MATHEMATICAL MODELLING IN 3D DYNAMICS OF A RIGID INTERACTING WITH A MEDIUM

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A generalization to the 3D case of plane-parallel motion of a solid interacting with resistant medium, for which the conditions of jet or escape flow hold, is considered. Because of using experimental information on the properties of jet flow, we are forced to consider a whole class of dynamic systems that possess the property of relative structural stability. An example of a dynamic system that has a complete set of first integrals expressible via elementary functions is given. The present paper is devoted to the study of the possible extension to the 3D case of results of plane-parallel dynamics of motion of a solid in resistant medium for which the conditions of jet or escape flow hold. Here the procedures of construction of three-dimensional phase portraits for systems with variable dissipation are applied. We consider an example of using these procedures in the study of a class of 3D motions of a solid in a resistant medium for which the system is subjected to a nonintegrable servoconstraint that makes it possible to consider the system of dynamical motion equations to be of smaller dimension.

Dynamic model of interaction of a rigid body with resisting medium provided jet flow, considered in activity [1,2], not only allows successfully to transfer outcomes appropriate problems from plane dynamics of a rigid body interacting with the medium and to receive their spatial analogs, but also to detect new cases of integrability till the Jacobi. Thus in some cases the integrals express through elementary functions. In activity the integrability of classical is shown in the problems about a spherical pendulum, located in a flow by filling of a medium, about spatial motion of a body at availability constraint, and also the mechanical and topological analogies are shown in the last two problems.

The hypothesizes concerning of properties of a medium, have found the reflection in construction of spatial (3D) dynamic model of interaction of a rigid body with resisting medium. In this connection there is a capability of formalizing of the model suppositions and obtaining of a full system of ordinary differential equations.

All interaction of medium with a body is concentrated on that part of a surface of a body which has the shape of convex plane area P.

As the interaction happens under the laws of jet flow the force S of this interaction is directed on a normal line to area and the point N of the acting of this force is determined only in one parameter - by an angle of attack α which is measured between vector of velocity v of a point D of a plate and external normal line in this point (straight line CD). The point D is the interception of the straight line CD (C - center of mass) that is perpendicular to plane P. Thus, $DN = R(\alpha)$.

Size of force of resistance we shall accept as $S = sv^2$, where v is the module of speed of a point D, and coefficient of resistance s is the function only of angle of attack: $s = s(\alpha)$.

There is the additional force T, which acts on a body on the straight line CD. Let's name it as "force of a thrust". The introduction of this force is used, as for maintenance of some specific classes of motions (thus T is the reaction of the possible (or probable) imposed constraint and in the methodical purposes, which pursue learning of interesting non-linear systems (having character of pendulum) arising at the reduction of the order. In case of absence external force T the body makes free braking (deceleration) in a resisting medium [3,4].

Systems of coordinates connected to a body shall designate through $Dxyz$. The last coordinate system connected to a point D is selected such that the tensor of inertia in the given system has diagonal type: $diag\{A, B, C\}$. Mass distribution we shall accept by such that longitudinal principal axis of inertia coincides an axis CD (it is an axis Dx), and the axes Dy and Dz lie in a plane P and will derivate with the right of coordinate system. Moreover, we shall consider case dynamically symmetrical rigid body, i.e. the equality $B = C$ is executed.

References

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