

# 2<sup>nd</sup> INTERNATIONAL CONFERENCE ON MATHEMATICAL MODELLING IN APPLIED SCIENCES

BSU Belgorod-Russia (August 20-24, 2019)



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# BOOK of Abstracts

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**Editor**

**Amar Debbouche, Guelma University, Algeria**

# Preface ICMMAS 2019

The *2nd International Conference on Mathematical Modelling in Applied Sciences, ICMMAS'19*, is organized by Belgorod State University (BSU), Belgorod-Russia, during the period August 20-24, 2019 <http://icmmas19.alpha-publishing.net/>. Its first version, named as ICMMAS'17, was held at SPbPU, Saint Petersburg-Russia in July 24-28, 2017 <http://icmmas.alpha-publishing.net/>. The evaluation committee within a meeting headed by Prof. V. Antonov and including Prof. A. Debbouche, Prof. D. Baleanu, Prof. S. Capozziello, Prof. V.E. Fedorov, Prof. W. Sproessig, Prof. D.F.M. Torres and Prof. I. Area witnessed the success of the scientific event providing some positive facts, and heartily recommended to repeat it every two years.

The proposed Scientific Program of the conference is including plenary lectures, contributed oral talks, poster sessions and listeners. Five suggested special sessions/mini-symposium are also considered by the scientific committee. The areas of interest include but are not limited to:

## TOPICS

**Models Based on Analytical, Numerical and Experimental Solutions**  
**Mathematical Modelling involving time fractional PDEs**  
**Biological Systems and Cancer Dynamics**  
**Ordinary and Partial Differential Equations: Theory and Applications**  
**Integral Equations and Integral Transforms**  
**Uncertainty Quantification in Mathematical Modelling**  
**Control Theory, Optimization and their Applications**  
**Probability, Statistics and Numerical Analysis**  
**Inverse Problems: Modelling and Simulation**  
**Modern Fractional Dynamic Systems and Applications**  
**Computational Methods in Sciences and Engineering**  
**Heat and Mass Transfer in Fractal Medium**

ICMMAS'19 is also supported by the following high ranked well known international journals:

### 1. Journal of Computational and Applied Mathematics

<https://www.journals.elsevier.com/journal-of-computational-and-applied-mathematics>

Impact Factor: 1.883

Guest Editors: Amar Debbouche, Michal Fečkan and Eduardo Hernández

### 2. Mathematical Methods in the Applied Sciences

[http://onlinelibrary.wiley.com/journal/10.1002/\(ISSN\)1099-1476](http://onlinelibrary.wiley.com/journal/10.1002/(ISSN)1099-1476)

Impact Factor: 1.533

Guest Editors: Amar Debbouche and Vladimir Vasilyev

### 3. Chaos, Solitons & Fractals

<https://www.journals.elsevier.com/chaos-solitons-and-fractals>

Impact Factor: 3.064

Guest Editors: Juan Carlos Cortés, Amar Debbouche and Sakthivel Rathinasamy

### 4. The European Physical Journal Plus

<https://www.springer.com/physics/applied+%26+technical+physics/journal/13360>

Impact Factor: 2.612

Guest Editors: Amar Debbouche, Juan J. Nieto and Delfim F.M. Torres

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# Integrable Dissipative Dynamical Systems: Approach and Applications

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**Abstract.** We study nonconservative systems for which the usual methods of the study, e.g., Hamiltonian systems, are inapplicable. Thus, for such systems, we must “directly” integrate the main equation of dynamics. We generalize previously known cases and obtain new cases of the complete integrability in transcendental functions of the equation of dynamics of a rigid body of different dimensions in a nonconservative force field.

**Keywords.** dissipative dynamical system, integrability, transcendental first integral.

**MSC2010.** 37C; 37E; 37L; 37N.

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We obtain a series of complete integrable nonconservative dynamical systems with nontrivial symmetries. Moreover, in almost all cases, all first integrals are expressed through finite combinations of elementary functions; these first integrals are transcendental functions of their variables. In this case, the transcendence is understood in the sense of complex analysis, when the analytic continuation of a function into the complex plane has essentially singular points. This fact is caused by the existence of attracting and repelling limit sets in the system (for example, attracting and repelling focuses). We detect new integrable cases of the motion of a rigid body, including the classical problem of the motion of a multi-dimensional spherical pendulum in a flowing medium.

This activity is devoted to general aspects of the integrability of dynamical systems with variable dissipation. First, we propose a descriptive characteristic of such systems. The term “variable dissipation” refers to the possibility of alternation of its sign rather than to the value of the dissipation coefficient (therefore, it is more reasonable to use the term “sign-alternating”) [1, 2].

We introduce a class of autonomous dynamical systems with one periodic phase coordinate possessing certain symmetries that are typical for pendulum-type systems. We show that this class of systems can be naturally embedded in the class of systems with variable dissipation with zero mean, i.e., on the average for the period with respect to the periodic coordinate, the dissipation in the system is equal to zero, although in various domains of the phase space, either energy pumping or dissipation can occur, but they balance to each other in a certain sense. We present some examples of pendulum-type systems on lower-dimension manifolds from dynamics of a rigid body in a nonconservative field.

Then we study certain general conditions of the integrability in elementary functions for systems on the two-dimensional plane and the tangent bundles of a one-dimensional sphere (i.e., the two-dimensional cylinder) and a two-dimensional sphere (a four-dimensional manifold). Therefore, we propose an interesting example of a three-dimensional phase portrait of a pendulum-like

system which describes the motion of a spherical pendulum in a flowing medium (see also [2, 3]).

To understand the difficulty of problem resolved, for instance, let us consider the spherical pendulum ( $\psi$  and  $\theta$  — the coordinates of point on the sphere where the pendulum is defined) in a jet flow. Then the equations of its motion are

$$\ddot{\theta} + (b_* - H_1^*)\dot{\theta} \cos \theta + \sin \theta \cos \theta - \dot{\psi}^2 \frac{\sin \theta}{\cos \theta} = 0, \quad (1)$$

$$\ddot{\psi} + (b_* - H_1^*)\dot{\psi} \cos \theta + \dot{\theta} \dot{\psi} \frac{1 + \cos^2 \theta}{\cos \theta \sin \theta} = 0, \quad b_* > 0, \quad H_1^* > 0, \quad (2)$$

and the phase pattern of the eqs. (1), (2) is on the Fig. 1.

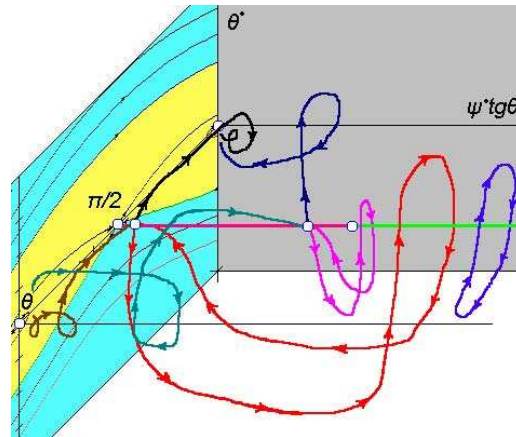


Figure 1: Phase pattern of spherical pendulum in a jet flow

The assertions obtained in the work for variable dissipation system are a continuation of the Poincaré–Bendixon theory for systems on closed two-dimensional manifolds and the topological classification of such systems.

The problems considered in the work stimulate the development of qualitative tools of studying, and, therefore, in a natural way, there arises a qualitative variable dissipation system theory.

## Acknowledgments

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