1. The solution to the well-known Kirchoff problem does not exhaust all possibilities of modelling the movement of a solid body in the boundless volume of an ideal incompressible fluid which is at rest at infinity. This is the case when the translational movement of the body in a medium is associated with a rotational one. Because of this, we are concerned with the problem of a plane-parallel movement when the direction of force that the medium exerts on the body is constant with respect to the body, its only change being the parallel translation dependent on the angle of attack. These conditions characterize the movement of a plate in a medium with a large angle of attack and the jet or discontinuous flow-past [1, 2].

Due to the cyclic character of some variables, a dynamical system of the sixth order makes it possible to split an independent subsystem of the third order [3]. Moreover, an independent subsystem of the second order can be considered, if the natural parameter along the phase trajectory is introduced as a new independent variable (see [3] for details).

With respect to quasi-velocities, the dynamic system takes the following final form:

$$\begin{aligned} \alpha' &= \omega + \frac{\sigma}{I}\mathcal{F}(\alpha)\cos\alpha + \sigma\omega^2\sin\alpha + \frac{s(\alpha)}{m}\sin\alpha, \\ \omega' &= -\frac{1}{I}\mathcal{F}(\alpha) - \omega\Psi(\alpha, \omega), \\ v' &= v\Psi(\alpha, \omega), \\ \varphi' &= \omega \\ x' &= \cos(\alpha - \varphi), \\ y' &= \sin(\alpha - \varphi), \end{aligned} \qquad (1)$$

where $(\ldots)' = \frac{d}{dq}(\ldots)$;

$$\Psi(\alpha, \omega) = -\sigma\omega^2\cos\alpha + \frac{\sigma}{I}\mathcal{F}(\alpha)\sin\alpha - \frac{s(\alpha)}{m}\cos\alpha$$

(q is a natural parameter). Here, σ, m, and I are the physical constants.

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The phase variables α (the angle of attack), ω (the angular velocity), v (modulus of the velocity of a body point), φ (rotation angle), x and y (the Cartesian coordinates of a body point) together form a stratifying six-dimensional phase space

$$\mathbf{S}^1\{\alpha \bmod 2\pi\} \times \mathbf{R}^1\{\omega\} \times \mathbf{R}^1_+\{v\}$$
$$\times \mathbf{S}^1\{\varphi \bmod 2\pi\} \times \mathbf{R}^2\{x, y\}.$$

The first two equations of the system form an independent subsystem of the second order. Let us denote it as (1).

The dynamic system contains the variables $\mathcal{F}$ and s. We qualitatively describe them using the experimental data on the properties of the jet flow-past. The following properties hold:

$$\mathcal{F} \in \Phi, \quad s \in \Sigma. \qquad (2)$$

Here, Φ and Σ are the physically admissible classes of sufficiently smooth functions. The class Φ consists of odd π-periodic functions that satisfy the following conditions: $\mathcal{F}(\alpha) > 0$ for $\alpha \in (0, \pi/2)$ and $\mathcal{F}'(0) > 0$, $\mathcal{F}'(\pi/2) < 0$. The class Σ consists of even 2π-periodic functions that satisfy the conditions $s(\alpha) > 0$ for $\alpha \in (0, \pi/2)$, $s(\alpha) < 0$ for $\alpha \in (\pi/2, \pi)$, and $s(0) > 0$, $s'(\pi/2) < 0$, $s(\alpha + \pi) = -s(\alpha)$.

The following conditions hold:

$$\begin{aligned} \mathcal{F} &= \mathcal{F}_0(\alpha) = AB\sin\alpha\cos\alpha \in \Phi, \\ s &= s_0(\alpha) = B\cos\alpha \in \Sigma, \quad A, B > 0. \end{aligned} \qquad (3)$$

2. The relative structural stability. Let us consider a deformation of a vector field over a subclass $\chi(\mathcal{K}) \subseteq \chi(C^r)$, $r \geq 1$, rather than over the entire set of fields [4]. Here, $\chi(C^r)$ denotes the smooth vector fields of the class C^r. The subclass $\chi(\mathcal{K})$ is defined with the aid of a functional class $\mathcal{K}$.

Definition. Let us call a system of differential equations that define a sufficiently smooth vector field $\mathbf{v}$ a relatively structurally stable (relatively coarse) system, or relatively coarse with respect to the class of functions $\mathcal{K} \subseteq C^r$, if any vector field $\mathbf{w}$ obtained by a sufficiently small deformation of the field $\mathbf{v}$ in the C^1-topology with respect to the functional class $\mathcal{K}$ is topologically equivalent to the field $\mathbf{v}$.

System (1) under condition (2) and certain other conditions has a family of phase portraits that do not change the topological type when the functions $\mathcal{F}$ and s are deformed along the classes Φ and Σ, respectively. Under these conditions, system (1) is structurally stable relative to the classes of functions Φ and Σ. In addition, any vector field that corresponds to the classes of these functions is structurally stable (coarse).

Thus, consideration of system (1) may be reduced to the consideration of the same system under condition (3). The corresponding system of equations is analytical.

3. The phase trajectories of the third-order system in the space $\mathbf{R}^2\{\alpha, \omega\} \times \mathbf{R}^1_+\{v\}$ lie on the two-dimensional cylindrical surfaces. Accordingly, the phase portrait of the system in $\mathbf{R}^2\{\alpha, \omega\} \times \mathbf{R}^1_+\{v\}$ is constructed on the basis of the phase portrait of system (1) in $\mathbf{R}^2\{\alpha, \omega\}$. The phase portrait on the plane is not a subset of the portrait in $\mathbf{R}^2\{\alpha, \omega\} \times \mathbf{R}^1_+\{v\}$ in the set-theoretical sense; rather, it is an orthogonal projection of the three-dimensional phase portrait onto the plane $\mathbf{R}^2\{\alpha, \omega\}$. Due to the cylindrical character of the vector field, the phase trajectories may be lifted from the plane into the space $\mathbf{R}^2\{\alpha, \omega\} \times \mathbf{R}^1_+\{v\}$ and the three-dimensional phase portrait can be obtained in this way.

4. Let us introduce the following notation:

$$\max_{\alpha \in (0,\pi)} \frac{\mathcal{F}(\alpha)}{\sin\alpha \cos\alpha} = g^*, \quad \frac{g^*}{I} = n^2,$$

$$\max_{\alpha \in (-\pi,\pi)} \frac{s(\alpha)}{\cos\alpha} = B$$

(the definition is continuously supplemented in the points $\alpha = \pm\pi/2$). Consider the plane $\mathbf{R}^2\{\alpha, \omega\}$. In system (1), there are two dimensionless parameters $\mu_1 = 2\frac{B}{mn}$, $\mu_2 = \sigma n$. The two-parametric infinite family of phase portraits corresponds to the following parameter domain:

$$\left\{ (\mu_1, \mu_2) \in \mathbf{R}^2 : \frac{s(\alpha)}{m}\sin\alpha \geq \frac{\sigma}{I}\mathcal{F}(\alpha)\cos\alpha,\ \forall\alpha \in \mathbf{R} \right\}. \tag{4}$$

Thus, we shall consider system (1) in the parameter domain (4).

Lemma 1.

(1) *The rest points* $(\pi k, 0)$, $k \in \mathbf{Z}$ *are repellers.*

(2) *The rest points* $(\pi/2 + \pi k, 0)$, $k \in \mathbf{Z}$ *are saddle-points.*

(3) *The rest points* $(\pm\pi/2 + 2\pi k, \mp 1/\sigma)$, $k \in \mathbf{Z}$ *are attractors.*

(4) *There are no other rest points in system* (1).

Lemma 2. *Under conditions* (2), (4), *the trajectories of system* (1) *do not form closed curves, both contractible to a point and not, on the phase cylinder* $\mathbf{S}^1\{\alpha \bmod 2\pi\} \times \mathbf{R}^1\{\omega\}$.

The key problem in the phase portrait classification is the behavior of stable and unstable separatrices that belong to hyperbolical saddlepoints.

Let us introduce the following bands on the phase plane:

$$\Pi_{(\alpha_1,\alpha_2)} = \{ (\alpha, \omega) \in \mathbf{R}^2 : \alpha_1 < \alpha < \alpha_2 \}.$$

In this case

$$\Pi_{(-\pi/2,\pi/2)} = \Pi; \quad \Pi_{(\pi/2, 3\pi/2)} = \Pi'.$$

Lemma 3. *An unstable separatrix corresponding to the domain* Π *and the point* $(-\pi/2, 0)$ *has the coordinate origin as its* α*-limiting set.*

Corollary. *Due to the central symmetry of the field, the behavior of the other analogous separatrices is similar to that described above.*

Lemma 4. *If condition* (4) *holds, the separatrix that enters the point* $(+\pi/2, 0)$ *in the* Π'*-band, has the point* $(\pi, 0)$ *as its* α*-limiting point.*

Corollary. *All analogous separatrices behave in the manner described above.*

Let us consider a key problem: the behavior of the separatrices that leave the points $\pi/2 \bmod 2\pi$, 0) and $(-\pi/2 \bmod 2\pi, 0)$.

Definition. The index of the separatrix behavior (subsequently denoted isp) is called the pair $\mathbf{k} = (\mathbf{k}_1, \mathbf{k}_2)$, where $\mathbf{k}_1 \in \mathbf{N}_0 \cup \{l + 1/4, l \in \mathbf{N}_0\}$. In this case,

$$\mathbf{k}_2 = \begin{cases} \text{either} & \mathbf{k}_1 - (1/2)\ (\mathbf{k}_1 \neq 0); \\ \text{or} & \mathbf{k}_1 + 1/2; \\ \text{or} & \mathbf{k}_1 - 1/4, \text{ if } \mathbf{k}_1 \in \mathbf{N}, \\ \text{and} & \mathbf{k}_2 = \mathbf{k}_1 + 1/4, \text{ if } \mathbf{k}_1 \notin \mathbf{N}_0. \end{cases}$$

By definition, isp $= \mathbf{k} = (\mathbf{k}_1, \mathbf{k}_2)$ if the separatrix that leaves the point $(-\pi/2, 0)$ has the point, separated from $(-\pi/2, 0)$ by a distance $2\pi\mathbf{k}_1$ along the α-axis, as its ω-limiting set, while the separatrix that leaves the point $(\pi/2, 0)$ has the ω-limiting set separated from $(\pi/2, 0)$ by a distance $2\pi\mathbf{k}_2$.

Note. This definition uses the fact that separatrices that leave the points $(\pi/2 + 2\pi l, 0)$, $l \in \mathbf{Z}$ lie within the domain bounded by the separatrices that leave the points $(-\pi/2 + 2\pi l, 0)$, $l \in \mathbf{Z}$.

Main Theorem. *For any* isp $= \mathbf{k}$ *from the domain of its definition, there exist separatrices that realize the corresponding behavior. Thus,* isp *can take any value in the domain where it is defined.*

Corollary. *Let us define a correspondence*

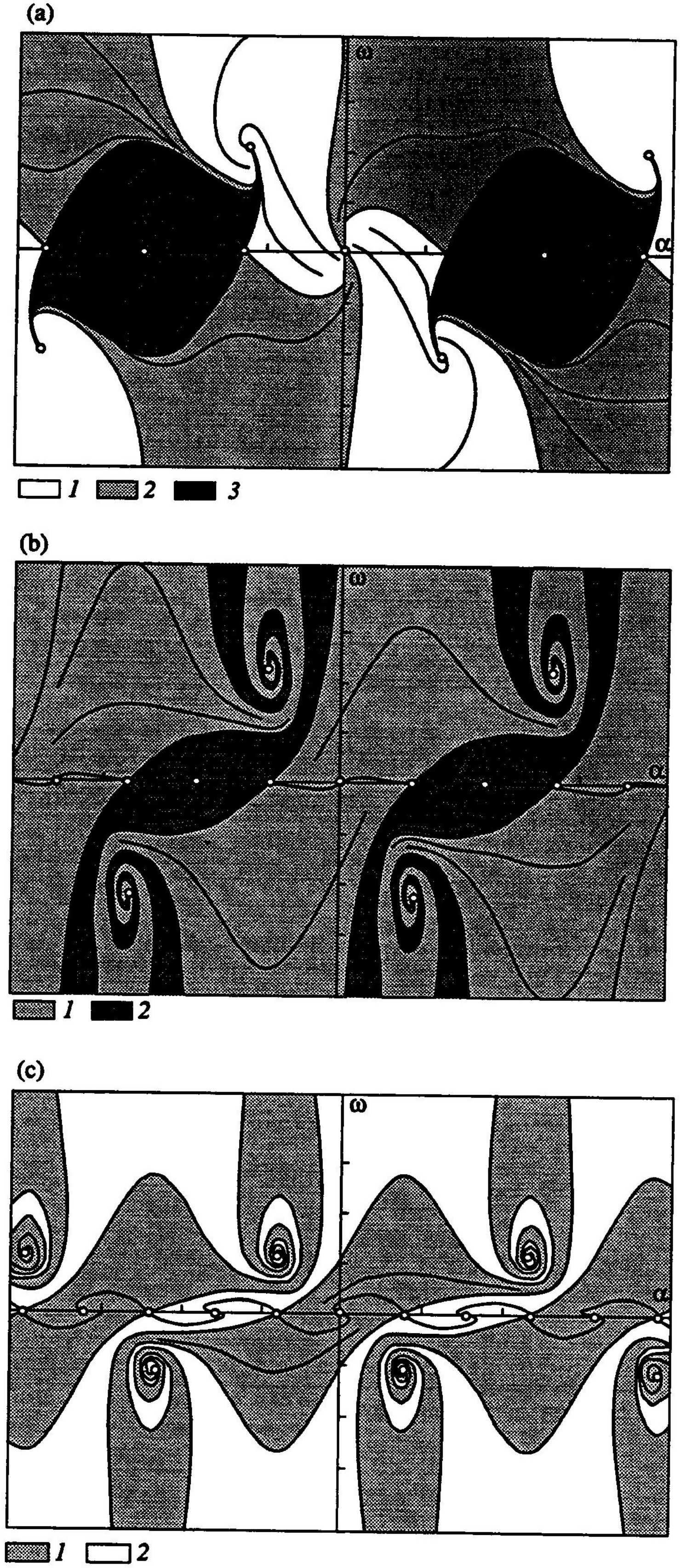

Fig. 1. Two-parametric families; the index of separatrix behavior equals (a) (0, 1/2); (b) (1/4, 1/2); and (c) (1, 1/2). (a): *1* and *2* are the unbounded domains with infinite limiting points, *3* is the bounded domain with limiting points in the finite region. (b), (c): *1* is the domain with the coordinate origin as the limiting set, *2* is the domain with symmetrical points as the limiting set.

isp ⇒ [*the behavior of the separatrices*].

This map is one-to-one.

This fact leads us to the following conclusion: any sufficiently small perturbation of the form (1) for the mathematical pendulum changes the global topological type of the phase portrait infinitely many times. Some of the portraits, with the index isp assuming the three lowest values, are shown in Fig. 1. The subsequent admissible values of isp are (1, 3/4), (1, 3/2), (5/4, 3/2), (2, 3/2) ... .

Thus, we can carry out the full topological classification of the phase portraits of system (1) under conditions (2) and (4). This justifies using the two-parametric family of phase portraits of the dynamic system to describe the movement of a body in a resistant medium. The family obtained consists of an infinite number of topologically different phase portraits.

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