The cases of complete integrability in dynamics of a rigid body interacting with a medium

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Some hypotheses on the properties of a medium are embodied in the three–dimensional (3D) dynamic model of interaction of bodies with a medium. All of the interaction of a medium with a body is concentrated on that part of the surface of the body which has the form of a convex plane region P.

Since the interaction is governed by the laws of jet flow: the force $\mathbf{S}$ of this interaction is directed along the normal to this region P; moreover, the point at which this force is applied is determined by both the angle of attack and the angular velocity. The latter is measured as the angle made by the velocity vector $\mathbf{v}$ of the point D of the plane plate and the outer normal at this point (the straight line CD, C is the center of mass).

The additional force $\mathbf{T}$ may act on the body along the straight line CD. As before, this force is called the driving force. This force is introduced to ensure the execution of certain specified classes of motions in a resisting medium (here, $\mathbf{T}$ is the reaction of possible imposed constraints). Under the absence of the external force $\mathbf{T}$, the body executes a 3D free deceleration (braking) in the resisting medium.

We consider a class of motions of a system that is constrained in such a way that the velocity of D can be assumed to be constant throughout the whole time of motion. We carry out a complete qualitative analysis of the obtained dynamical system in the quasi–velocity space. The symmetries in the systems are pointed out, and an explicit formula for the first integral as a transcendental function of quasi–velocities is presented. We also consider the possibilities of extending the results of plane dynamics of rigid bodies interacting with a medium to the 3D case, analyze the motion of a spherical pendulum placed into the incoming flow of a medium and the 3D motion of bodies under the existence of servo–constraints and also demonstrate mechanical and topological analogs of the last two problems [1].

New integrable cases and families of phase portraits in the plane dynamics of rigid bodies are discovered. Certain model cases of the motion of rigid bodies in a resisting medium are qualitatively studied and integrated. The first integrals of corresponding systems are found; these integrals are transcendental functions and functions that can be expressed in terms of elementary functions. New families of multidimensional phase portraits in the spatial dynamics are found. The problem of the dynamically symmetrically fixed rigid body placed in the flow of an incoming medium is integrated.

[1] M. V. Shamolin, Methods of Analysis of Dynamical Systems with Various Dissipation in Dynamics of a Rigid Body. Ekzamen Press, Moscow, 2007, pp. 1–352.