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New cases of integrability in dynamics of a rigid body with the cone form of its shape interacting with a medium

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The purpose of the activity is to elaborate the qualitative methods for studying the dynamics of rigid bodies interacting with a resisting medium under quasistationarity conditions. This material refers equally to the qualitative theory of ordinary differential equations and the dynamics of rigid bodies. We use the properties of body's motion in a medium under conditions of the jet flow past this body. We study the plane model problems of the motion of a body with the cone form of its shape in a resisting medium. The new families of phase portraits of variable dissipation systems are obtained, their absolute or relative roughness is demonstrated. The integrable cases of equations of motion of rigid bodies are found.

1 Introduction

This paper contains some the development of qualitative methods in the theory of nonconservative systems that arise, e.g., in such fields of science as the dynamics of a rigid body interacting with a resisting medium, oscillation theory, etc. This material can call the interest of specialists in the qualitative theory of ordinary differential equations, in rigid body dynamics, as well as in fluid and gas dynamics since the work uses the properties of motion of a rigid body in a medium under the streamline flow around conditions.

The author obtains a complete integrability cases for nonconservative dynamical systems having nontrivial symmetries. Moreover, in almost all cases of integrability, each of the first integrals is expressed through a finite linear combination of elementary functions and is a transcendental function of its variables, simultaneously. In this case, the transcendence is meant in the complex analysis case, i.e., after the continuation of the functions considered to the complex domain, they have essentially singular points. The latter fact is stipulated by the existence of attracting and repelling limit sets in the system considered.

2 Cone in a jet flow

Assume that a axe-symmetric homogeneous rigid body of mass m executes a plane-parallel motion in a medium with quadratic resistance law (see below) and that a certain part of the exterior body surface has a cone form being under the medium streamline flow conditions [1]. This means that the action of the medium on the plate reduces to the force $\mathbf{S} = \mathbf{S_x} + \mathbf{S_y}$ (applied at the point N, Fig. 1). Let the remained par of the body surface be situated in a volume bounded by the flow surface that goes away from the plate boundary and is not subjected by the medium action. For example, similar conditions can arise after the body entrance into the water.

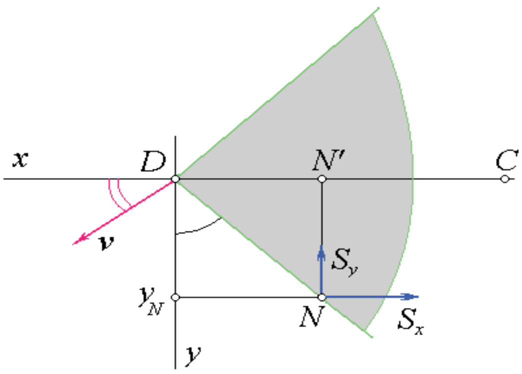

Fig. 1 Cone in a jet flow.

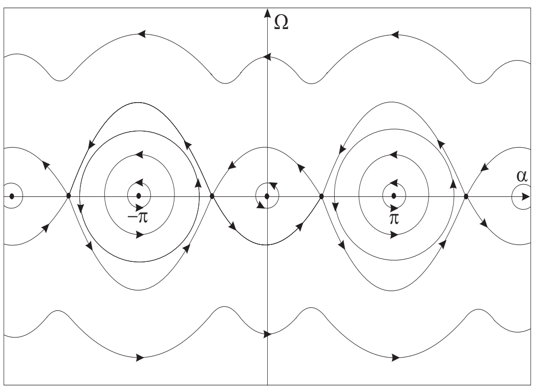

Fig. 2 One of the possible types of conservative phase patterns of comparison for the system investigated.

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3 Dynamical part of the equations of the rigid body motion

Assume that among the body motions, there exists a rectilinear plane-parallel drag regime. This is possible when the following condition hold the body velocity is on the axe of symmetry CD (C is the center of mass, Fig. 1). Let us relate to the body the right coordinate system Dxy too, and let us introduce the following phase coordinates: the value $v = |\mathbf{v}|$ of the velocity $\mathbf{v}$ of the point D, the angle of attack α between the vector $\mathbf{v}$ and the axis x, and the absolute angular velocity Ω of the body. We assume that $y_N = y_N(\alpha),\ x_N = -DN',\ CN' = \sigma - DN',\ \sigma = DC$. Furthermore we assume that $\mathbf{S_x} = -s(\alpha)v^2\mathbf{e_x},\ \mathbf{S_y} = -b(\alpha)v^2\mathbf{e_y}$ too. Then the dynamical part of the equations of the body motion has the following form (I is the central moment of inertia):

$$\dot v\cos\alpha - \dot\alpha v\sin\alpha - \Omega v\sin\alpha + \sigma\Omega^2 = F_x/m,\ \dot v\sin\alpha + \dot\alpha v\cos\alpha + \Omega v\cos\alpha - \sigma\dot\Omega = F_y/m$$

$$I\dot\Omega y_N s(\alpha)v^2 - (\sigma - DN')b(\alpha)v^2,\ F_x = -S_x,\ F_y = -S_y$$

It is important to note that we accept the following functions as the forms of S. A. Chaplygyn [1, 2]: $-x_N = -x_N(\alpha) = DN' = A_2\mathrm{sgn}\alpha\sin\alpha,\ y_N = y_N(\alpha) = A_1\sin\alpha,\ A_1, A_2 > 0,\ s(\alpha) = B_1\cos\alpha,\ b_(\alpha) = B_2\sin\alpha,\ B_1, B_2 > 0.$

If we consider more general problem of a body motion in a resisting medium when the controlled force $\mathbf{T}$ acts to the body (along the line CD) with the above force of resistance, and gets the fulfillment of the following condition

$$v = \mathrm{const}$$

then $F_x = T - S_x$, and the last condition is the equivalence. Thus, the order of the dynamical part of the equations of the body motion equals two and the own systems is equivalent almost eweryehere the following system:

$$\dot\alpha = -\Omega + \frac{\sigma v}{I}(A_1B_1\sin\alpha + A_2B_2\mathrm{sgn}\alpha\tan\alpha\sin\alpha - \sigma B_2\tan\alpha) - \frac{B_2 v}{m}\tan\alpha$$

$$\dot\Omega = \frac{v^2}{I}(A_1B_1\sin\alpha + A_2B_2\mathrm{sgn}\alpha\tan\alpha\sin\alpha - \sigma B_2\tan\alpha)$$

It is interesting to note that even the corresponding comparison conservative system for the system considered

$$\dot\alpha = -\Omega,\ \dot\Omega = \frac{v^2}{I}(A_1B_1\sin\alpha + A_2B_2\mathrm{sgn}\alpha\tan\alpha\sin\alpha - \sigma B_2\tan\alpha)$$

has the types of the phase patterns, one of them is illustrated in the Fig. 2.

4 On the integrability of the system considered

Let us correspond the system considered of the second order the following nonautonomous equation ($\tau = \sin\alpha$):

$$\frac{d\Omega}{d\tau} = \frac{v^2\Psi(\tau,\cos\alpha)/I}{-\Omega + \sigma v\Psi(\tau,\cos\alpha)/I - B_2 v\tau/m},\ \Psi(\tau,\cos\alpha) = A_1B_1\tau\cos\alpha + A_2B_2\mathrm{sgn}\tau\tau^2 - \sigma B_2\tau$$

Theorem 4.1 *Last equation has the solution can be expressed in terms of transcendental (in sense of complex analysis) functions* [2, 3].

5 Conclusion

The results of the presented work were appeared owing to the study the applied problem of the rigid body motion in a resisting medium, where we have obtained the transcendental first integral expressed through a finite combination of elementary functions. This circumstance allows the author to carry out the analysis of all phase trajectories and show those their properties which have the roughness and are preserved for systems of a more general form. The complete integrability of such system is related to their symmetries of latent type. Therefore, it is of interest to study a sufficiently wide class of dynamical systems having analogous latent symmetries.

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