New families of many-dimensional phase portraits in dynamics of a rigid body interacting with a medium

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Abstract

In this activity the 3D motion of the body in a resisting medium is considered. It is famous that if it is interesting to investigate the motion of some rigid body in a medium it is necessary to consider both problem of the motion of a rigid body and the motion of a medium. Some method of the description of the motion of the rigid body in terms of not partial differential equations and ordinary ones is suggested. The dynamic system arising is depending of the whole class of differentiable functions. Therefore it is actual both the problem of (absolute) structural stability and relative structural stability too.

Key words: phase portrait, rigid body, resisting medium.

AMS subject classifications: 70E99, 58F40.

1 Introduction

Let us illustrate briefly next problem of a rigid body motion in a resisting medium. Our rigid body has a simple shape which has a plate as a part of the surface. The plane plate is the most simple body which permits to investigate various features of motion in a medium. Effects connected with influence of attached weights (classical problem of Kirchhoff) are demonstrated in a text-book of Lamb [1]. Let's remark, that the given problem of the Kirchhoff was begun in the second half of the last century arises the interesting aspect of consideration of such problems. Such aspect is connected to the problems of integrability of a non-linear system differential equations. The given problems deal with the existence of analytical (smooth, meromorphic) first integrals.

Let us indicate also another aspect of consideration of that problem, and just, on the qualitative analysis of systems of differential equations, describing the given motion (stratification of a phase space, its topology, qualitative description of phase trajectories, symmetry and etc.). And though the listed problems are connected to integrability closely, their solution has independent interest and has independent character. Moreover, the last aspect forms and stimulates the development of the qualitative vehicle. For example, at the general suppositions about character of aerodynamic effects let us consider the problems of existence both structural stability of motion of a rigid in a resisting medium with a jet flow. A special shape of surface of a body and hypothesis about quasi-static effect of a medium have allowed to formulate the full scheme of forces, in which mass, the geometrical and aerodynamic characteristics are entered.

Mathematical model, used in activity, ascends to activities of Vitaly A. Samsonov, Boris Ya. Lockshin, and Vladimir A. Privalov [2, 3]. So in some activities the phase portrait of physical pendulum located in a flow of a medium with a jet flow is presented. It is interesting that such dynamic system describing the motion of a pendulum has rather nontrivial non-linear properties. And also the problems about the structural stability of some fixed straight-line (rectilinear) motions of a free body in a medium in a jet flow are considered. The research is conducted on the base of non-linearized equations of the motion of a body.

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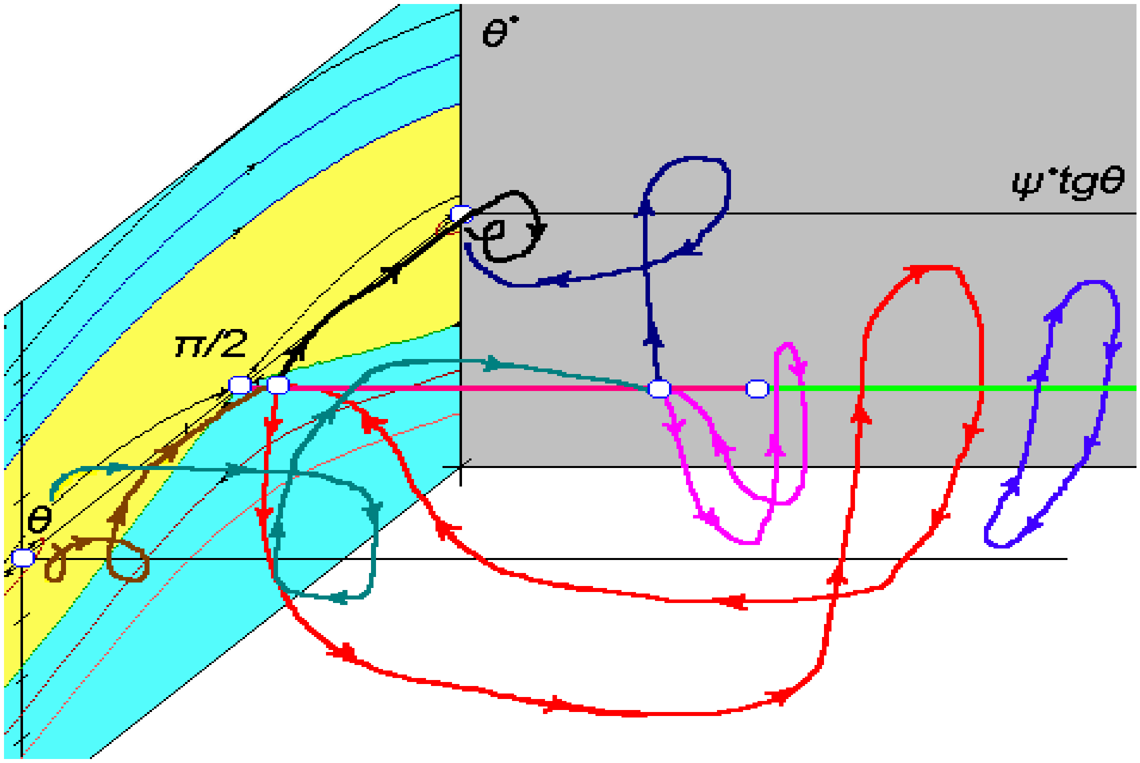

Figure 1: Sample of possible phase portraits.

2 Equations of motion in some cases

For example let us consider the motion of dinamical symmetric (I_1, $I_2 = I_3$ are the main moments of inertia) rigid body and ($F(\alpha)$, $s(\alpha)$) is a pair of functions which "answers" for the aerodynamic interaction of a rigid body with a resisting medium under the assumptions of a jet flow [2, 3] (α is an angle of attack). The force of resistance is directed to an axe of dynamical symmetry. Therefore, the component of the angular velocity ω_{x0} in the projection to that axe is the first integral.

If there the Z_1, Z_2 are some functions of the rest of components of the angular velocity, (v, α, β) – are the spherical coordinates of the velocity of some point of a rigid body, σ is some geometrical constant, m is a mass of a body, that the dinamic differential equations of the spartial deceleration of a rigid body in a medium have next form (q is a natural parameter along the trajectory of some point):

$$
\begin{aligned}
\frac{dv}{dq} &= \psi_1(\alpha, Z_1, Z_2)v + \sigma\frac{I_1}{I_2}\omega_{x0}Z_1\sin\alpha \\
\frac{d\alpha}{dq} &= -Z_2 + \sigma(Z_1^2 + Z_2^2)\sin\alpha + \frac{\sigma}{I_2}F(\alpha)\cos\alpha + \frac{\sigma I_1}{vI_2}\omega_{x0}Z_1\cos\alpha + \frac{s(\alpha)}{m}\sin\alpha \\
\frac{dZ_2}{dq} &= \frac{1}{I_2}F(\alpha) + Z_2[-\psi_1(\alpha, Z_1, Z_2) - \frac{\sigma I_1}{vI_2}\omega_{x0}Z_1\sin\alpha] - Z_1\psi_2(v, \alpha, Z_1, Z_2) \\
\frac{dZ_1}{dq} &= Z_1[-\psi_1(\alpha, Z_1, Z_2) - \frac{\sigma I_1}{vI_2}\omega_{x0}Z_1\sin\alpha] + Z_2\psi_2(v, \alpha, Z_1, Z_2) \\
\frac{d\beta}{dq} &= Z_1\frac{cos\alpha}{\sin\alpha} - \frac{\omega_{x0}}{v} + \frac{\sigma I_1}{vI_2}\omega_{x0}\frac{Z_2}{\sin\alpha}
\end{aligned} \tag{1}
$$

where $\psi_1(\alpha, Z_1, Z_2) = -\sigma(Z_1^2 + Z_2^2)\cos\alpha + \frac{\sigma}{I_2}F(\alpha)\sin\alpha - \frac{s(\alpha)}{m}\cos\alpha$, $\psi_2(v, \alpha, Z_1, Z_2) = -\frac{I_1}{I_2}\frac{\omega x0}{v} + Z_1\frac{\cos\alpha}{\sin\alpha} + \frac{\sigma I_1}{vI_2}\omega_{x0}\frac{Z_2}{\sin\alpha}$. Some of the possible types of the phase portraits is presented on the fig. 1.

3 Some remarks about structure of the phase portraits

And so, in the main system (1) of differential equations describing the spartial rigid body motion in a resisting medium under assumption $\omega_{x0} = 0$ the system is depending of quasi–velocities of order three is independent. There is such non-trivial and interesting family of nonequivalent phase portraits in spartial Dynamics. The degree of difficulty in that case is higher and there are a lots of problems of qualitative character which are required the generalization of both famous classical methods and the new methods obtained. For example:

Examples 3.1

1. *The existence and uniqueness the trajectories which have the infinite points as their limited sets in three-dimensional spaces.*

2. *The existence of closed orbits which present the non-trivial component of the fundamental group of three-dimensional manifold in what the dinamic system is considered.*

3. *A lots of problems of the behavior of separatrixes which have the saddle points as their limited sets.*

The quantity of such types of portraits of the system (1) under assumption $\omega_{x0} = 0$ is is not finite. Therefore, we have obtained the non-trivial (small as well as possible) perturbation of the classical (mathematical) pendulum such that the topological type of classical pendulum is changed by the infinite quantity of times in any (small as well as possible) neighborhood of non-perturbated parameters.

References

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