Structural Stability in 3D Dynamics of a Rigid

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1. Abstract

In this activity the 3D motion of the body in a resisting medium is considered. It is famous that if it is interesting to investigate the motion of some rigid body in a medium it is necessary to consider both problem of the motion of a rigid body and the motion of a medium. It is suggested some method of the description of the motion of the rigid body in terms of not partial differential equations and ordinary ones. The dynamic system arising is depending of the whole class of differentiable functions. Therefore it is actual both the problem of (absolute) structural stability and relative structural stability too. As an example the interesting family of phase portraits is shown. Such family obsesses of some non-trivial non-linear properties.

2. Keywords

Structural stability, phase portrait, 3D rigid body dynamics, resisting medium

3. Some general view-point

Let's illustrate briefly next problem of a rigid motion in a resisting medium. Our rigid body has a simple shape which has a plate as a part of the surface.

It is plane plate - most simple body is permitting to investigate various features of motion in a medium. Effects connected with influence of attached weights (classical problem of Kirchhoff) are demonstrated in a text-book of Lamb [1].

Let's remark, that the given problem of the Kirchhoff was begun in the second half of the last century arises the interesting aspect of consideration of such problems. Such aspect is connected to the problems of integrability of a non-linear system of differential equations. The given problems deal with the existence of analytical (smooth, meromorphic) first integrals. And in general, concept of integrability (and nonintegrability) in dynamics of a rigid body interacting with a medium "answers" for complicated (or for simple) behavior of trajectories in a phase space, connected with chaos.

Let's indicate also another aspect of consideration of an indicated problem, and just, on the qualitative analysis of systems of differential equations, describing the given motion (stratification of a phase space, its topology qualitative description of phase trajectories, symmetry and etc.). And though the listed problems closely are connected to integrability, their solution has independent interest and has independent character. Moreover, the last aspect forms and stimulates the development of the qualitative vehicle.

For example, at the general suppositions about character of aerodynamic effects let us consider the problems of structural stability of motion of a rigid in a resisting medium with a jet flow. It is interesting also to consider the problem of structural stability of permanent rotation of a rigid in a flow of a medium. A special design of surface of a body and hypothesis about quasi-static effect of a medium have allowed to formulate the full scheme of forces, in which mass, the geometrical and aerodynamic characteristics are entered.

Mathematical model, used in activity, of motion of a rigid body ascends to activities of Vitaly A. Samsonov, Boris Ya. Lockshin, and Vladimir A. Privalov [2,3]. So in some activities the phase portrait of physical pendulum located in a flow of a medium with a jet flow. It is interesting that such dynamic system describing the motion of a pendulum has rather nontrivial non-linear properties. And also in this activity the problems about the structural stability of some fixed straight-line (rectilinear) motions of a free body in a medium in a jet flow are considered. The research is conducted on base of non-linearized equations of motion of a body.

In general, the problem of research of motion of a body in a field of force of resistance "rests" on absence of the full description, of given the field of forces. As it is known, it is measure resistance basically in fixed experiment. And here making field of forces adequate the quasi-velocities of a system arises only in non-stationary motion of a body. Therefore the process of the description of a field of force are represented by the sequence of the steps. Some preliminary model of a field of force at first is created the set of mechanical systems. And here it is actual to consider the problem of the structural stability of the classes of motion. And also it is constructed the set of various features which essentially dependent on those parameters of model that the information about is not full or is absent completely. In an outcome of research of such model there are the problems and the answers on which within the framework of adopted model cannot be found.

4. About the problem

Let us consider the 3D rigid body motion in a resisting medium under assumption of that the jet flow is fulfilled. The general system of ordinary differential equations permits the reduction of non-depended system of the order six [4,5].

After that by the famous way of the reduction of the general order [6] it is possible to consider the non-depended system of ordinary differential equations of the order three.
After that we obtain (in 3D case) the system is depending of two classes of the smooth functions as it is considered the experimental information about the properties of a jet flow by the medium. Even in 2D case we have the non-trivial family of the phase patterns. In this case the system looks like the following system of the order two:

$$\alpha' = \Omega + \frac{\sigma}{I}\sin\alpha\cos^2\alpha + \sigma\Omega^2\sin\alpha + \frac{B}{m}\sin\alpha\cos\alpha$$

$$\Omega' = -\frac{1}{I}\sin\alpha\cos\alpha - \Omega\Psi(\alpha,\Omega)$$

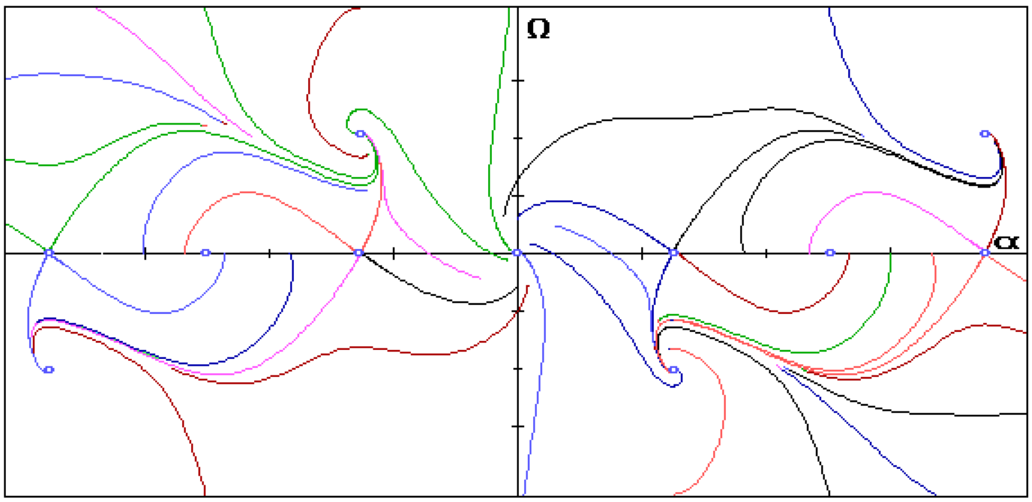

Fig. 1

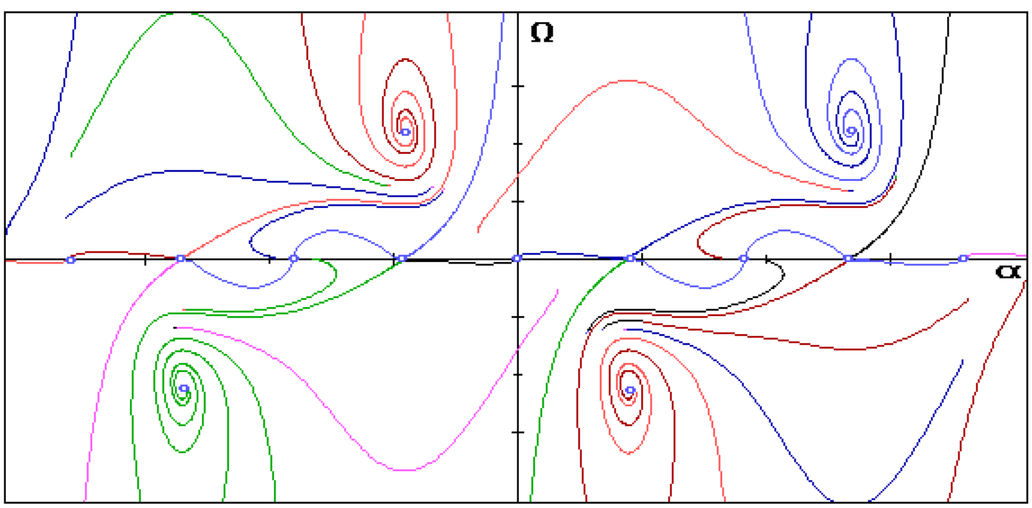

Fig. 2

where

$$\Psi(\alpha,\Omega)=\frac{\sigma}{I}\sin^2\alpha\cos\alpha-\sigma\Omega^2\cos\alpha-\frac{B}{m}\cos^2\alpha$$

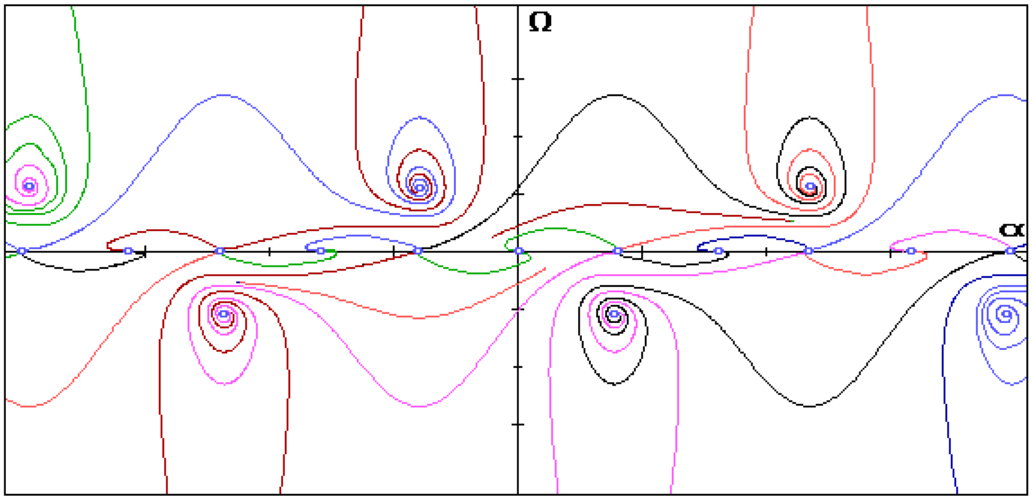

Fig. 3

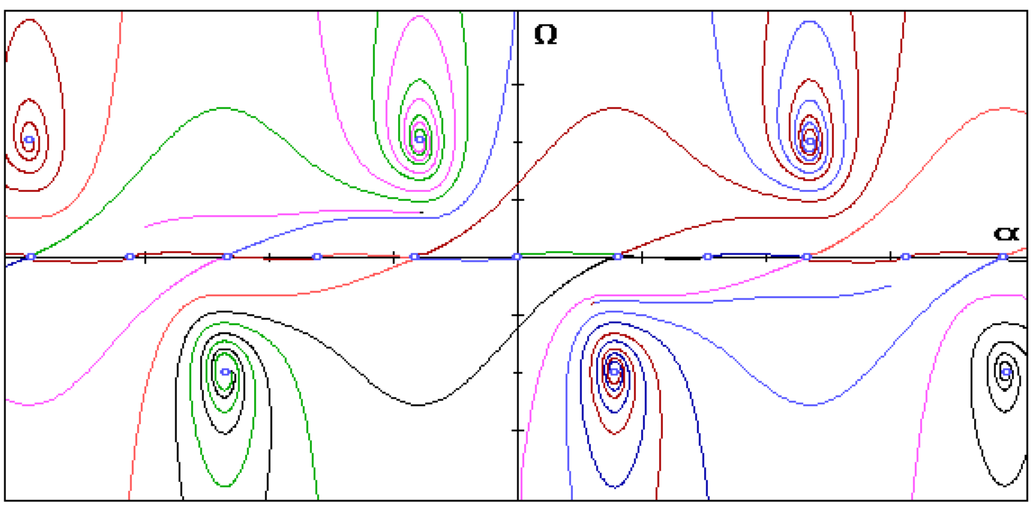

Fig. 4

5. Figures

Some of the interesting portraits are presented on the figs.1-10. The main difference between the topological types of portraits presented is an so-called index of separatrix behavior. Such index codes the definite type of the portrait. To obtain this way of coding of the portraits it needs to develop some classical methods of qualitative theory of ordinary differential equations on the smooth manifolds and "restrict" such methods to dissipative systems or to the systems with the various dissipation.

It is interesting and important to notice that not all the portraits presented are (absolute) structural stable. Some of portraits are not (absolute) structural stable and are relatively structural stable phase portraits. Such portraits are the structural stable in sense of its deformation not in all class of functions and only in some subclass which is the subset of the set of all class of the smooth vector field on the two-dimensional cylinder.

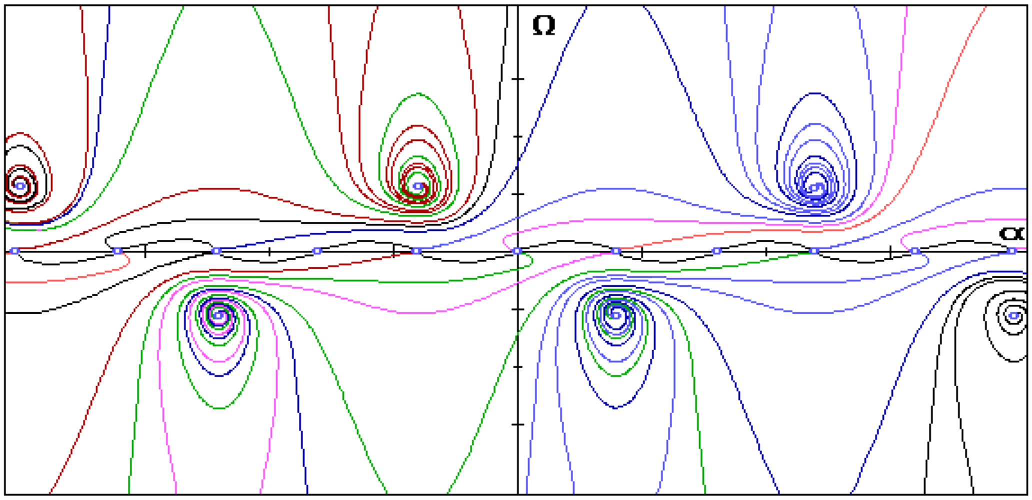

Fig. 5

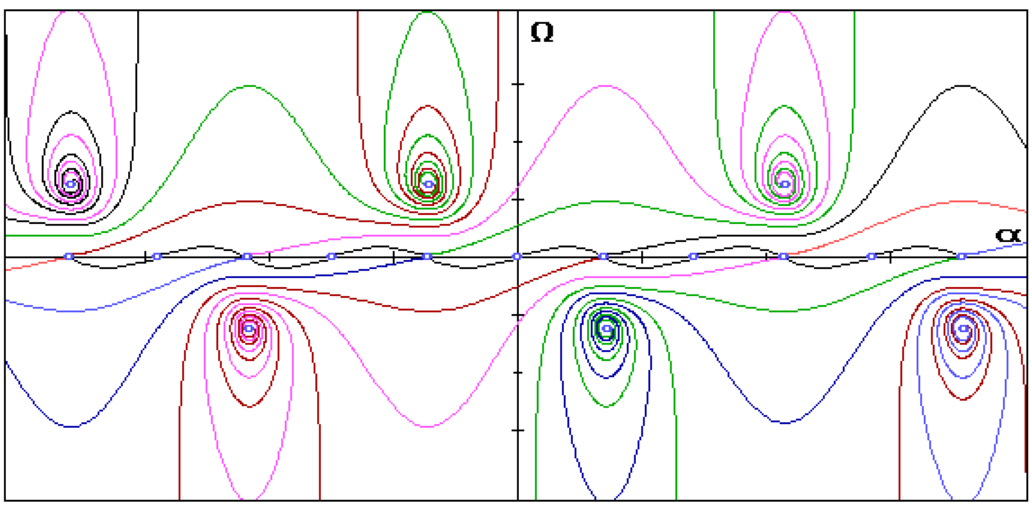

Fig. 6

In the mail system of differential equations describing the three-dimensional rigid body motion in a resisting medium the system of quasi-velocities of order three is independent. There is such non-trivial and interesting family of non-equivalent phase portraits in 3D Dynamics. The degree of difficulty in that case is higher and there are lots of problems of qualitative character which are required the generalization of both famous classical methods and the new methods obtained.

The last group of methods is required to develop next problems:

- the existence and uniqueness the trajectories which have the infinite points as their limited sets in three-dimensional spaces;
- the existence of closed orbits which present the non-trivial component of the fundamental group of three-dimensional manifold in what the dinamic system is considered;
- a lots of problems of the behavior of separatrixes which have the saddle points as their limited sets.

The last problem is most complicated and required the development of the new methods of the qualitative theory of the ordinary differential equations on the smooth three-dimensional manifolds.

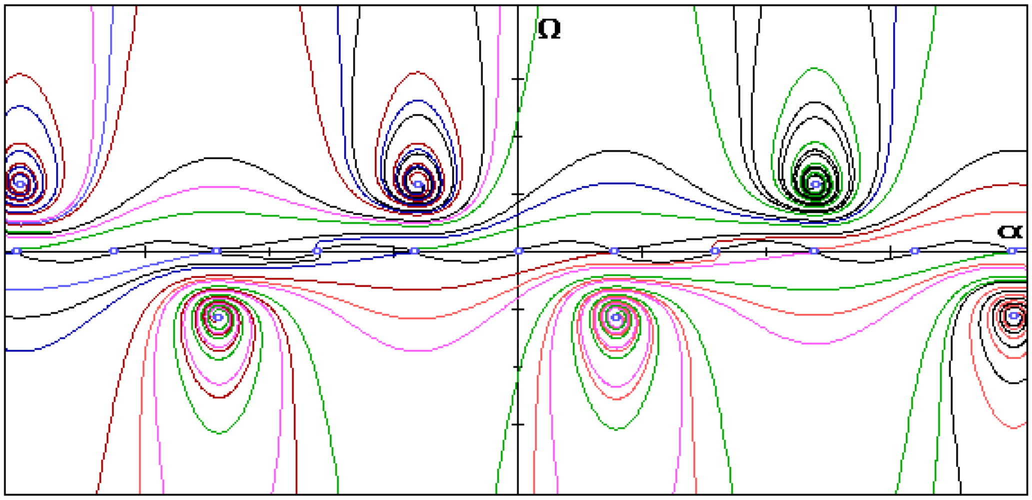

Fig. 7

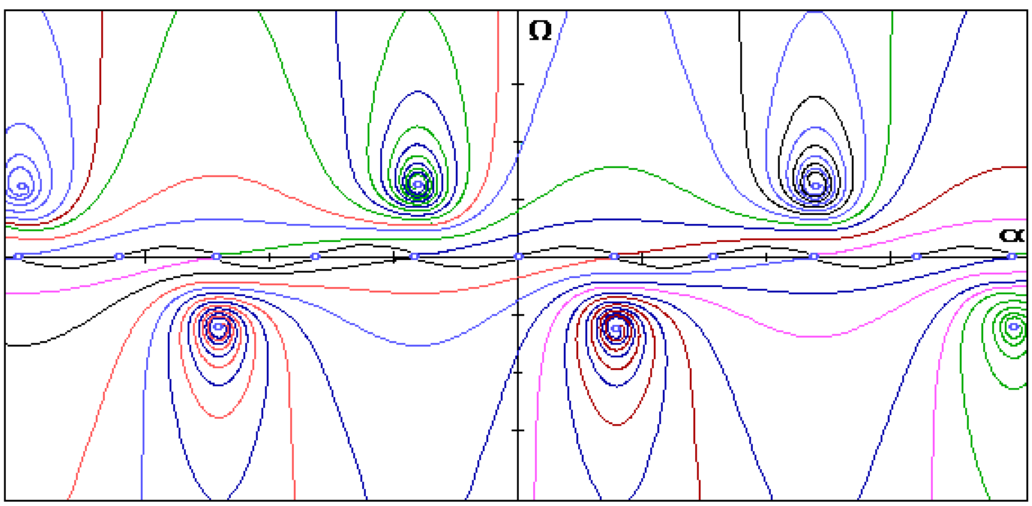

Fig. 8

It is easy to understand that the quantity of such types of portraits is not finite. Therefore, we have obtained the non-trivial (small as well as possible) perturbation of the classical (mathematical) pendulum such that the topological type of classical pendulum is changed by the infinite quantity of times in any (small as well as possible) neighborhood of non-perturbated parameters of the following system

$$\alpha' = \Omega, \ \Omega' = -\frac{1}{I} \sin\alpha \cos\alpha$$

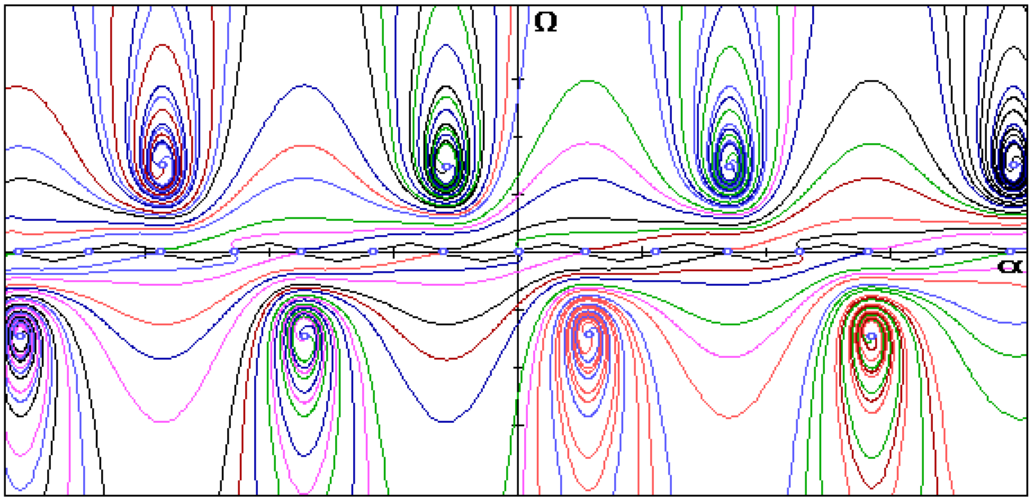

Fig. 9

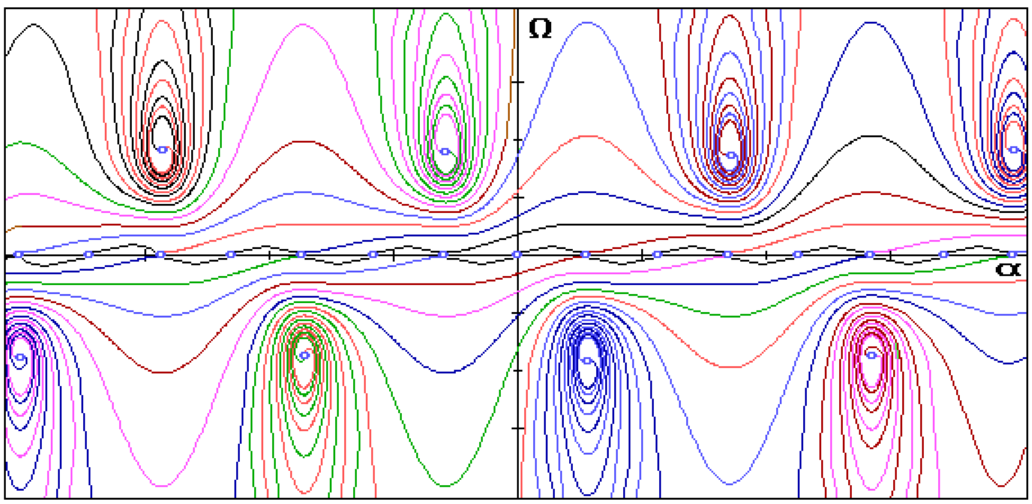

Fig. 10

6. References

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