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Azərbaycan Milli Elmlər Akademiyası
Riyaziyyat və Mexanika İnstitutu

Riyaziyyat və Mexanikanın Müasir Problemləri

Riyaziyyat və Mexanika İnstitutunun
60-illik yubileyinə həsr olunmuş beynəlxalq konfransın

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Modern Problems of Mathematics and Mechanics

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60th anniversary of the Institute of Mathematics and Mechanics
of Azerbaijan National Academy of Sciences

Современные Проблемы Математики и Механики

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Национальной Академии Наук Азербайджана

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INTEGRABLE DISSIPATIVE DYNAMICAL SYSTEMS: BACKGROUNDS, METHODS, AND APPLICATIONS

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We study nonconservative systems for which the usualmethods of the study, e.g., Hamiltonian systems, are inapplicable.Thus, for such systems, we must “directly” integrate the mainequation of dynamics. We generalize previously known cases andobtain new cases of the complete integrability in transcendentalfunctions of the equation of dynamics of a rigid body of differentdimensions in a nonconservative force field.

We obtain a series of complete integrable nonconservative dynamicalsystems with nontrivial symmetries. Moreover, in almost all cases,all first integrals are expressed through finite combinations ofelementary functions; these first integrals are transcendentalfunctions of their variables. In this case, the transcendence isunderstood in the sense of complex analysis, when the analyticcontinuation of a function into the complex plane has essentiallysingular points. This fact is caused by the existence of attractingand repelling limit sets in the system (for example, attracting andrepelling focuses). We detect new integrable cases of the motion ofa rigid body, including the classical problem of the motion of amulti-dimensional spherical pendulum in a flowing medium.

This activity is devoted to general aspects of the integrability ofdynamical systems with variable dissipation. First, we propose adescriptive characteristic of such systems. The term “variabledissipation” refers to the possibility of alternation of its signrather than to the value of the dissipation coefficient (therefore,it is more reasonable to use the term “sign-alternating”) [1, 2].

We introduce a class of autonomous dynamical systems with oneperiodic phase coordinate possessing certain symmetries that aretypical for pendulum-type systems. We show that this class ofsystems can be naturally embedded in the class of systems withvariable dissipation with zero mean, i.e., on the average for theperiod with respect to the periodic coordinate, the dissipation inthe system is equal to zero, although in various domains of thephase space, either energy pumping or dissipation can occur, butthey balance to each other in a certain sense. We present someexamples of pendulum-type systems on lower-dimension manifolds fromdynamics of a rigid body in a nonconservative field [2, 3].

Then we study certain general conditions of the integrability inelementary functions for systems on the two-dimensional plane andthe tangent bundles of a one-dimensional sphere (i.e., thetwo-dimensional cylinder) and a two-dimensional sphere (afour-dimensional manifold). Therefore, we propose an interestingexample of a three-dimensional phase portrait of a pendulum-likesystem which describes the motion of a spherical pendulum in aflowing medium (see also [1, 4, 5]).

The assertions obtained in the work for variable dissipation systemare a continuation of the Poincare–Bendixon theory for systems onclosed two-dimensional manifolds and the topological classificationof such systems.

The problems considered in the work stimulate the development ofqualitative tools of studying, and, therefore, in a natural way,there arises a qualitative variable dissipation system theory.

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ON FRAME PROPERTIES OF ITERATES OF A MULTIPLICATION OPERATOR

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Dynamical sampling that is a relatively new research topic in applied harmonic analysis has attracted considerable attention in recent years (see, for example, [1] and the bibliography therein). One of the central problems in dynamical sampling is investigation of frame properties for families of elements obtained by iterates of operators.

This note is dedicated to the study of frame properties of iterates of a multiplication operator $T_\varphi f(t) = \varphi(t)\cdot f(t),\ f \in L_2(a,b)$.

The following theorem is obtained in [1]:

Theorem 1. *Let $\varphi(t)$ be any measurable function and $f(t)$ any square summable function on (a,b). The system $\{T_\varphi^n f\}_{n=0}^{\infty}$ cannot be a frame in $L_2(a,b)$.*