შოთა რუსთაველის ეროვნული სამეცნიერო ფონდი
SHOTA RUSTAVELI NATIONAL SCIENCE FOUNDATION

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საერთაშორისო სამეცნიერო კონფერენცია

უწყვეტ გარემოთა მექანიკის მონათესავე პრობლემები

International Scientific Conference

Related Problems of Continuum Mechanics

მოხსენებათა კრებული
PROCEEDINGS

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საერთაშორისო სამეცნიერო კონფერენცია

უწყვეტ გარემოთა მექანიკის მონათესავე პრობლემები

ეძღვნება აკაკი წერეთლის სახელმწიფო უნივერსიტეტის დაარსებიდან 85 წლისთავს

ეძღვნება პროფესორ ავთანდილ თვალჭრელიძის
ხსოვნას და დაბადებიდან 70 წლისთავს

International Scientific Conference

Related Problems of Continuum Mechanics

Dedicated to 85th Anniversary of the AkakiTsereteli State University

The Conference is dedicated to the memory
of Professor Avtandil Tvalchrelidze and his 70th birth anniversary

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კონფერენცია „უწყვეტ გარემოთა მექანიკის მონათესავე პრობლემები“ ტარდება საგრანტო პროექტის „მათემატიკური მოდელი და კომპიუტერული პროგრამები მრავალშრიანი ტექსტილის მრავალფუნქციური კვლევისათვის'' ფარგლებში. პროექტი ხორციელდება აკაკი წერეთლის სახელმწიფო უნივერსიტეტის მიერ შოთა რუსთაველის ეროვნული სამეცნიერო ფონდის ფინანსური მხარდაჭერით.

კრებულში წარმოდგენილია სტატიები შემდეგი სექციების მიხედვით:

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კონფერენციის კრებულში შევიდა 37 მოხსენება, რომლებიც წარმოდგენილია მეცნიერების მიერ მსოფლიოს 7 ქვეყნიდან.

იმედს გამოვთქვამთ, რომ აღნიშნული კონფერეცია ხელს შეუწყობს საქართველოს და სხვა ქვეყნების ორგანიზაციების სამეცნიერო თანამშრომლობის განვითარებას უწყვეტ გარემოთა მექანიკის საკითხებში.

Preface

Conference on Related Problems of Continuum Mechanics is held within project “Mathematical Models and Computer Program for Multi-functional Researches of Multi-layer Textile Composites”. Project is implementing by the Akaki Tsereteli State University (Kutaisi, Georgia) under financial support of the Shota Rustaveli National Science Foundation (Georgia).

This book includes the papers covering the following topics:

- Mechanics of Thin Bodies
- Mathematical Modeling of Multilayered Structures
- Applied Problems of Continuum Mechanics
- Related Problems of Analysis

Proceeding of the Conference includes 37 papers of scientists from 7 countries.

We do hope that this Conference will support the scientific relations between Georgian and foreign institutions on continuum mechanics fields.

მიხეილ ნიქაბაძე, კონფერენციის სამეცნიერო კომიტეტის თავმჯდომარე

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Oscillations During Spatial Deceleration of a Rigid Body in a Resisting Medium

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Abstract

Some qualitative analysis is carried out of the spatial problems concerning the motion of a rigid body in a resisting medium. A nonlinear model is constructed of impact of the medium on the rigid body, which takes into account the dependence of the arm of force on the reduced angular velocity of the body. Moreover, the moment of this force itself is also a function of the angle of attack. As was shown by the processing the experimental data on the motion of homogeneous circular cylinders in water, these circumstances should be taken into account in the simulation. The analysis of the plane and spatial models of the interaction of a rigid body with a medium reveal the sufficient conditions of stability of the key regime of motion, i.e., the translational rectilinear deceleration. It is also shown that, under certain conditions, both stable or unstable auto-oscillating regimes can be presented in the system.

Keywords: rigid body, resisting medium, translational deceleration

1 Introduction

Under study is the problem of motion of a rigid body interacting with a medium only through the flat front portion of its outer surface. In constructing the force action of the medium, we use information on the properties of the jet flow around and assume the quasi-stationary conditions [1]. The motion of the medium is not inspected, but we consider such a problem of the dynamics of a rigid body in which the characteristic time of the body motion with respect to its center of mass is commensurate with the characteristic time of motion of the center itself.

Owing to the complexity of nonlinear analysis, at the initial stage of our study we neglected the dependence of the mediums force moment on the angular velocity of the body and used the only dependence on the angle of attack [2, 3].

From a practical point of view, an important issue is studying the stability of the so-called unperturbed (rectilinear translational) motion for which the velocities of the body points are perpendicular to a flat portion (cavitator).

The whole range of results obtained under this simple assumption allows us to conclude that it is impossible to find the conditions under which the above systems would have solutions corresponding to the bodys angular oscillations of a bounded amplitude.

An experiment on the motion of homogeneous circular cylinders in water [3, 4] confirmed that, in simulating the influence of the medium on a solid, it is indeed necessary to take it into account that the moment of force of the medium also depends on the angular velocity of the body. In this case, some additional terms appear in the equations of motion, which introduce dissipation into the system.

When studying the motion of a body with finite angles of attack, the fundamental question of nonlinear analysis consists in the finding the conditions under which the oscillations of a bounded amplitude exist near the unperturbed motion, which confirms the need for a complete nonlinear analysis.

Note also that in this paper the author realizes the idea of a phenomenological quasi-stationary approach to the simulation of a body motion in some resisting medium. Surely, simulation of the impact of the medium on a rigid body is not limited to the approach applied in the present article. There are other purely non-stationary hydrodynamic mechanisms of resistance, different from the considered, for example, the damping connected with the effect of added masses [3, 4].

2 Spatial Motion of an Axisymmetric Rigid Body in a Resisting Medium

Let us consider the problem of spatial motion of a homogeneous axisymmetric rigid body of mass m, a part of the surface of which has the shape of a flat circular disk interacting with the medium according to the laws of jet flow around [2,3]. Suppose that the remaining part of the body surface is not affected by the medium and places inside the volume bounded by the jet surface breaking away from the edge of the disk. Similar conditions may arise, for example, after the entry of a homogeneous circular cylinder into water [4, 5].

Suppose that the tangential forces to the disk are absent. Then the force $\mathbf{S}$ applied to the body at the point N from the medium, does not change its orientation relative to the body (directed along the normal to the disk) and is quadratic in the velocity of its center D (Newtonian resistance, Fig. 1). It is also assumed that the force of gravity acting on the body is negligible as compared with the force of the mediums resistance (influence).

If these conditions are satisfied then, among the motions of the body, there is a mode of rectilinear translational deceleration similar to the case of rectilinear (unperturbed) motion: the body is able to perform translational motion in the direction of its axis of symmetry, i.e. perpendicular to the plane of the disk.

We associate with the body a right coordinate system $Dxyz$ (Fig. 1) and direct the Dx axis along the axis of geometric symmetry of the body. The axes Dy and Dz are rigidly connected with the circular disc forming a right coordinate system. The components of the angular velocity vector Ω in the system $Dxyz$ will be denoted by $\{\Omega_x, \Omega_y, \Omega_z\}$. The inertia tensor of the dynamically symmetric body in the introduced body axes $Dxyz$ has a diagonal form: $\mathrm{diag}\{I_1, I_2, I_2\}$.

We use the hypothesis of quasistationarity and assume for simplicity that the quantity $R_1 = DN$ is determined at least by the angle of attack α measured between the velocity vector $\mathbf{v}$ of the center D of the disk and the axis Dx. Thus, $DN = R_1(\alpha, \ldots)$.

We take the value of the resistance force in the form $S = |\mathbf{S}| = s_1(\alpha)v^2$, $v = |\mathbf{v}|$.

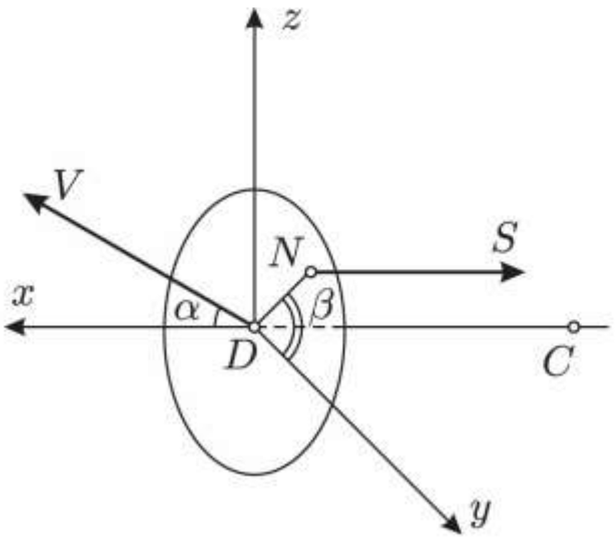

Figure 1: Spatial motion of axisymmetric rigid body in a resisting medium

For convenience of further description (as in the case of rectilinear motion), instead of the coefficient of resistance $s_1(\alpha)$, we introduce an auxiliary alternating-sign function $s(\alpha)$: $s_1 = s_1(\alpha) = s(\alpha)\mathrm{sgn}\cos\alpha > 0$. Thus, the pair of functions $R_1(\alpha, \ldots)$ and $s(\alpha)$ determines the force-moment characteristics of the medium influence on the disk under these model assumptions.

2.1 Dynamic Part of the Equations of Spatial Motion

Consider the spherical coordinates (v, α, β) of the end of the vector $\mathbf{v} = \mathbf{v}_D$ of the velocity of the point D with respect to the flow, in which the angle is measured in the plane of the disk (see Fig. 1). The quantities (v, α, β) are expressed by some nonintegrable relations through cyclic kinematic variables and their derivatives [2,3]. Therefore, we consider the triple (v, α, β) as quasi-velocities, adding to them the components $(\Omega_x, \Omega_y, \Omega_z)$ of the angular velocity in the axes associated with the body. It is obvious that with respect to these axes $\mathbf{v}_D = \{v\cos\alpha, v\sin\alpha\cos\beta, v\sin\alpha\sin\beta\}$.

By the theorems on the motion of the center of mass (in the projections onto the body axes $Dxyz$) and on the change in the kineticmoment with respect to these axes, we obtain the dynamic part of the differential equations of motion considered in the six-dimensional phase space of quasi-velocities (σ is the distance DC). The first group of equations corresponds to the motion of the center of mass itself, and the second, to the motion around the center of mass:

$$\begin{aligned}
&\dot v\cos\alpha - \dot\alpha v\sin\alpha + \Omega_y v\sin\alpha\sin\beta - \Omega_z v\sin\alpha\cos\beta + \sigma(\Omega_y^2+\Omega_z^2) = -\frac{s(\alpha)v^2}{m},\\
&\dot v\sin\alpha\cos\beta + \dot\alpha v\cos\alpha\cos\beta - \dot\beta v\sin\alpha\sin\beta + \Omega_z v\cos\alpha - \Omega_x v\sin\alpha\sin\beta -\\
&-\sigma\Omega_x\Omega_y - \sigma\dot\Omega_z = 0,\\
&\dot v\sin\alpha\sin\beta + \dot\alpha v\cos\alpha\sin\beta + \dot\beta v\sin\alpha\cos\beta + \Omega_x v\sin\alpha\cos\beta - \Omega_y v\cos\alpha -\\
&-\sigma\Omega_x\Omega_z + \sigma\dot\Omega_y = 0,\quad I_1\dot\Omega_x = 0,\\
&I_2\dot\Omega_y + (I_1 - I_2)\Omega_x\Omega_z = -z_N s(\alpha)v^2,\quad I_2\dot\Omega_z + (I_2 - I_1)\Omega_x\Omega_y = y_N s(\alpha)v^2,
\end{aligned} \tag{1}$$

where $(0, y_N, z_N)$ are the coordinates of the point N in the system $Dxyz$.

Now we construct the functional classes $\{y\}$, $\{s\}$. Considering the experimental information on the properties of jet flow around [1, 2], we formally describe these classes consisting of sufficiently smooth, 2π-periodic functions ($y(\alpha)$ is odd, while $s(\alpha)$ is even), satisfying the following conditions: $y(\alpha) > 0$ for $\alpha \in (0, \pi)$, and $y'(0) > 0$, $y'(\pi) < 0$ (class of functions $\{y\} = Y$); $s(\alpha) > 0$ for $\alpha \in (0, \pi/2)$, $s(\alpha) < 0$ for $\alpha \in (\pi/2, \pi)$, and $s(0) > 0$, $s'(\pi/2) < 0$ (class of functions $\{s\} = \Sigma$). Both y and s change sign under the substitution of α by $\alpha + \pi$. Thus,

$$y \in Y,\ s \in \Sigma. \tag{2}$$

Similar to the choice of the medium impact functions, we take the dynamical functions s, y_N, and z_N in the system (1) in the form (2), and also

$$y_N = R(\alpha)\cos\beta - h_1\frac{\Omega_z}{v},\ z_N = R(\alpha)\sin\beta + h_1\frac{\Omega_y}{v},$$

(in this case, the function R corresponds to the function y, cf. also with [2, 7]). In the system under consideration still there is an additional damping (and, in some domains of the phase space, accelerating) moment of the nonconservative force.

By (1), at all time there is cyclic invariant relation $\Omega_x \equiv \Omega_{x0} = \text{const}$. In what follows we will investigate the case of zero turning of a rigid body about its longitudinal axis; i.e., when the condition $\Omega_{x0} = 0$ is fulfilled.

Projecting further the angular velocities onto the moving axes not related to the body, so that $z_1 = \Omega_y\cos\beta + \Omega_z\sin\beta$, $z_2 = -\Omega_y\sin\beta + \Omega_z\cos\beta$, and introducing, as before, new dimensionless phase variables and differentiation by the formulas $z_k = n_0 v Z_k$, $k = 1, 2$, $<\cdot> = n_0 v <'>$, we reduce system (1) to the following form:

$$v' = v\Psi_1(\alpha, Z_1, Z_2), \tag{3}$$

$$\alpha' = -Z_2 + \mu_2(Z_1^2 + Z_2^2)\sin\alpha + \frac{\sigma}{I_2 n_0}F(\alpha)\cos\alpha - \frac{\sigma h_1}{I_2}Z_2 s(\alpha)\cos\alpha + \frac{s(\alpha)}{m n_0}\cos\alpha, \tag{4}$$

$$Z_2' = \frac{F(\alpha)}{I_2 n_0^2} - Z_2\Psi_1(\alpha, Z_1, Z_2) - Z_1^2\frac{\cos\alpha}{\sin\alpha} - \frac{\sigma h_1}{I_2}Z_1^2\frac{s(\alpha)}{\sin\alpha} - \frac{h_1}{I_2 n_0}Z_2 s(\alpha), \tag{5}$$

$$Z_1' = -Z_1\Psi_1(\alpha, Z_1, Z_2) + Z_1 Z_2\frac{\cos\alpha}{\sin\alpha} + \frac{\sigma h_1}{I_2}Z_1 Z_2\frac{s(\alpha)}{\sin\alpha} - \frac{h_1}{I_2 n_0}Z_1 s(\alpha), \tag{6}$$

$$\beta' = Z_1\frac{\cos\alpha}{\sin\alpha} + \frac{\sigma h_1}{I_2}Z_1\frac{s(\alpha)}{\sin\alpha}, \tag{7}$$

$$\Psi_1(\alpha, Z_1, Z_2) = -\mu_2(Z_1^2 + Z_2^2)\cos\alpha + \frac{\sigma}{I_2 n_0}F(\alpha)\sin\alpha - \frac{s(\alpha)}{m n_0}\cos\alpha - \frac{\sigma h_1}{I_2}Z_2 s(\alpha)\sin\alpha.$$

In the case of the Chaplygin functions [1, 6]

$$y(\alpha) = A\sin\alpha \in \{y\},\ A = y'(0) > 0,\ s(\alpha) = B\cos\alpha \in \{s\},\ B = s(0) > 0, \tag{8}$$

of the mediums influence, the analytic system of equations has the form

$$v' = v\Psi_1(\alpha, Z_1, Z_2), \tag{9}$$

$$\alpha' = -Z_2 + \mu_2(Z_1^2+Z_2^2)\sin\alpha + \mu_2\sin\alpha\cos^2\alpha - \mu_2\mu_3 Z_2\cos^2\alpha + \frac{\mu_1}{2}\sin\alpha\cos\alpha, \quad (10)$$

$$Z_2' = \sin\alpha\cos\alpha - Z_2\Psi_1(\alpha, Z_1, Z_2) - (1+\mu_2\mu_3)Z_1^2\frac{\cos\alpha}{\sin\alpha} - \mu_3 Z_2\cos\alpha, \quad (11)$$

$$Z_1' = -Z_1\Psi_1(\alpha, Z_1, Z_2) + (1+\mu_2\mu_3)Z_1 Z_2\frac{\cos\alpha}{\sin\alpha} - \mu_3 Z_1\cos\alpha, \quad (12)$$

$$\beta' = (1+\mu_2\mu_3)Z_1\frac{\cos\alpha}{\sin\alpha}, \quad (13)$$

$$\Psi_1(\alpha, Z_1, Z_2) = -\mu_2(Z_1^2+Z_2^2)\cos\alpha + \mu_2\sin^2\alpha\cos\alpha - \frac{\mu_1}{2}\cos^2\alpha - \mu_2\mu_3 Z_2\sin\alpha\cos\alpha,$$

where, as above, the dimensionless parameters μ_1, μ_2, and μ_3 are selected as follows:

$$\mu_1 = 2\frac{B}{mn_0},\ \mu_2 = b = \sigma n_0,\ n_0^2 = \frac{AB}{I_2},\ \mu_3 = H_1 = \frac{Bh_1}{I_2 n_0}.$$

Equations (4)–(7) (or (10)–(13)) of the system (3)–(7) (or (9)–(13)) form an independent subsystem of the fourth order, while the equations (4)–(6) (or (10)–(12)) of the third order.

2.2 On the Stability of Rectilinear Translational Deceleration

Let us investigate the stability of the key mode, the unperturbed motion, in relation to perturbations of the angle of attack and angular velocity, i.e., in relation to the variables α, Z_1, and Z_2. In other words, we investigate the stability of the trivial solution of the independent third-order system (4)–(6) (if, of course, we extend the definition of this system by continuity at the origin).

The following important proposition is valid:

Proposition 2.1. *The plane*

$$\{(\alpha, Z_1, Z_2) \in \mathbf{R}^3 :\ Z_1 = 0\} \quad (14)$$

is an integral surface for the system (4)–(6).

Furthermore, after the formal insertion $Z_1 = 0$ in (4)–(6), the remaining two equations for α and Z_2 form a system describing the dynamics of the rectilinear motion of the body [1].

Thus, the phase portrait of flat dynamics “falls” on the plane (14). Moreover, plane (14) separates the three-dimensional phase space into the two parts:

$$\{(\alpha, Z_1, Z_2) \in \mathbf{R}^3 :\ 0 < \alpha < \pi,\ Z_1 > 0\} \quad (15)$$

and $\{(\alpha, Z_1, Z_2) \in \mathbf{R}^3 :\ 0 < \alpha < \pi,\ Z_1 < 0\}$, and in each of them the motion occurs independently, but not arbitrarily in relation to each other because the system has the symmetry

$$\begin{pmatrix} \alpha \\ Z_1 \\ Z_2 \end{pmatrix} \to \begin{pmatrix} \alpha \\ -Z_1 \\ Z_2 \end{pmatrix}$$

with respect to the plane $\{(\alpha, Z_1, Z_2) \in \mathbf{R}^3 : \ Z_1 = 0\}$. The latest facts show that it suffices to investigate the system (4)–(6) in the semibounded layer (15), although it cannot be considered a full-fledged phase space.

An important consequence of the latest remarks is the possibility of using the function

$$V_1(\alpha, Z_1) = Z_1 \sin \alpha \qquad (16)$$

as a Lyapunov (Chetaev) function in the semibounded layer (15) since this function is positive definite in it.

Theorem 2.1. *The function (16) is a Lyapunov (Chetaev) function for the system (4)–(6), i.e., its derivative according to (4)–(6) is negative definite for $\mu_3 > \mu_1 + \mu_2$ and positive definite for $\mu_3 < \mu_1 + \mu_2$.*

Corollary 2.1. *For $\mu_3 > \mu_1 + \mu_2$ the system (4)–(6) has an attracting singular point at the origin (after the extension of definition of the right-hand sides in it), and when $\mu_3 < \mu_1 + \mu_2$ it has a repulsive point.*

Indeed, the derivative of (16) according to (4)–(6) is represented as

$$(\mu_1 + \mu_2 - \mu_3) Z_1 \alpha + \overline{o}(\alpha^2 + Z_1^2 + Z_2^2).$$

In particular, an analogous theorem holds also for the systems of the form (10)–(12) taken for the Chaplygin functions of the mediums influence [1, 6].

When we pass to the problem on the motion of homogeneous circular cylinders, we can conclude that this asymptotic stability holds under the fulfillment of the inequality

$$\sigma k + \frac{2I_2}{mD} < hD,$$

where D is the cylinder diameter, σ is the distance DC, whereas, k and h are the dimensionless parameters of the water influence on the cylinder, or

$$\sigma D k + 2r_1^2 < hD^2,$$

where r_1 is the radius of inertia of the cylinder.

It can be observed that Theorem 2.1 gives the same conditions for asymptotic stability with respect to the variables (α, Z_1, Z_2) as Proposition 2.1, which involves dynamical systems from the dynamics of rectilinear motion.

In the case of spatial motion, the resulting systems have an uncertainty at the origin, which is caused by the degeneracy of the spherical coordinates of the end of the vector $\mathbf{v}$, which is the velocity of the center of the front disk (cavitator). This uncertainty is overcome by the definition of the extension of the right-hand sides of dynamical systems.

3 Conclusion

Instability of the simplest movement of the body, the rectilinear translational deceleration, is used for methodological purposes, namely, for determination of unknown parameters of the environmental impact on a solid body under the conditions of quasistationarity.

An experiment on the motion of homogeneous circular cylinders in water, carried out at the Institute of Mechanics of Lomonosov Moscow State University, confirmed the fact that, in the simulation of the influence of the medium on a rigid body, it is necessary to take into account an additional parameter which introduces dissipation into the system.

When studying the class of deceleration of a body with finite angles of attack, the main issue is finding the conditions under which there exist auto-oscillations in a finite neighborhood of rectilinear translational deceleration. There arises, therefore, the need for a comprehensive nonlinear study.

The initial stage of this research is the neglecting the damping influence from the medium to the rigid body. In functional language, this means the assumption that the pair of dynamical functions that determine the effect of the medium depends only on one parameter, the angle of attack. The dynamical systems that arise in such a nonlinear description belong to the type of systems with variable dissipation. Therefore, there is a need to create a methodology of studying such systems (see also [2, 3]).

In the qualitative description of the interaction of the body with the medium, because of the use of experimental information on the properties of jet flow around, there arise a certain variation in the simulation of the force-moment characteristics. This makes natural the introduction of the definition of relative coarseness (relative structural stability) and the proof of such coarseness for the systems under study [3]. Moreover, many of the systems under consideration turn out to be just (absolutely) coarse in the sense of Andronov–Pontryagin in the ordinary sense.

In this article, some study is carried out of the motion of a body in a medium taking into account the damping moment from the medium. Such a moment introduces additional dissipation into the system; as a result, rectilinear translational deceleration, in principle, can become stable.

Thus, under certain conditions, taking into account the damping effect from the medium on the rigid body leads to a positive answer to the main question: during the motion of the body in a medium with finite angles of attack, in principle, the occurrence of stable auto-oscillations is possible.

If the parameters of the problem admit the presence of a critical case, then, depending on the top-order derivatives of the medium influence functions R and s, rectilinear translational deceleration of the body can be either stable, or unstable with respect to perturbations of the angle of attack and angular velocity.

In addition, sufficient conditions are found for such stability or instability, including inequalities on the top-order derivatives of the functions of the mediums influence. However, the main difficulty is the impossibility of experimentally measuring these derivatives in explicit form.

The work also provides some reasoning concerning the way one can study the behavior of a body near rectilinear translational deceleration (that is, stable or unstable angular oscillations) using experimental information, thereby implicitly estimating the top-order derivatives of the medium influence functions (see also [8–10]).

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