Data preparation for execution of experiments on rigid body motion in a resisting medium

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Summary. Proposed work presents next stage of the study of the problem of the plane-parallel motion of a rigid body interacting with a resistant medium through the frontal plane part of its external surface. Under constructing of the force acting of medium, we use the information on the properties of medium streamline flow around in quasi-stationarity conditions (for instance, on the homogeneous circular cylinder input into the water). The medium motion is not studied, and we consider such problem in which the characteristic time of the body motion with respect to its center of masses is comparable with the characteristic time of motion of the center of masses itself.

Preliminaries

From the practical view point it is important the problem of studying of stability of so-called unperturbed (rectilinear translational) motion under which the velocities of body points are perpendicular to the plate (cavitator).
The whole spectrum of results found under the simplest assumption on the absence of the medium damping action on a rigid body allowed the author to make the conclusion that it is impossible to find those conditions under which there exist the solutions corresponding to the angular body oscillations of a finite amplitude.
The experiment in the motion of homogeneous circular cylinders in the water (see [1, 2]) justified that in modelling the medium action on the rigid body, it is also necessary to take account of an dependence of the medium interaction force moment on the angular velocity of the body. Herewith, there arise the additional members that brings a dissipation to the system.
In studying the class of body motions with the finite angles of attack, the principal problem is finding those conditions under which there exist the finite amplitude oscillations in a neighborhood of the unperturbed motion. Therefore, there arises the necessity of a complete nonlinear study.
The account of the medium damping action on the rigid body leads to an affirmative answer to the principal question of the nonlinear analysis: under the body motion in a medium with finite angles of attack, in principle, there can arise stable auto-oscillations which can be explained by the account of an additional dependence of the medium action on the body angular velocity that brings an additional dissipation to the system.

Problem of Input of Homogeneous Circular Cylinders in Water

Further, let return to the problem of input of homogeneous circular cylinders in water (see [1, 2]). The values of physical parameters of cylinders, for which the rectilinear translational deceleration (drag) can be stable in principle, must be related by the relation

$$h\frac{mD^2}{I} - 2 - k\frac{m\sigma D}{I} > 0. \tag{1}$$

Herewith, if the value in the left-hand side of the inequality (1) is equal to zero then we deal with *critical case*.
Recall that D is the diameter of the circular cylinder, σ is the distance from the center of mass of a body to the front butt end, the constants I, m are the mass-inertia characteristics of cylinder, the constants k and h are the dimension-free parameters of medium action on the cylinder (see [1, 2]).
For the parameters k and h of the water action on the body with the front circular butt end, the estimates $k = h = 0,1$ have already been accepted. Thus the condition (1) allows to try to "construct" a rigid body (a circular cylinder) for which the rectilinear translational deceleration (drag) could be stable. For this, it takes to choose the parameters σ, D, I, m of a cylinder Starting from the condition (1).
Analyzing the inequality (1), we can conclude the following. The inertia-mass parameters of the homogeneous cylinders are such that inequality (1) is not possible to satisfy for $h = 0,1$.
The rectilinear translational deceleration (drag) of the homogeneous circular cylinder in the water *can not be stable* with respect to the perturbations of the angle of attack and angular velocity.
Nevertheless, we note that problem of stability studied can be resolved in accordance with the estimation on this coefficient accepted earlier: $h = 0,1$.

Problem of Input of Hollow Circular Cylinders in Water

Let set the problem for determining of geometrical and inertia-mass parameters of the combined rigid body, i.e., the hollow cylinder, for the possible attaining of such a stability. Namely, let imagine a certain hollow cylinder (a "cartridge", Fig. 1), the geometrical and inertia-mass characteristics of which, further, will allow to satisfy the desired conditions (see [1, 2]).
The combined rigid body studied is represented by the front homogeneous part (a cylinder) of the diameter D and the height $2\Delta_1$ which can be continued by the the lateral partitions of length $2\sigma_1$ and width Δ_2 (Fig. 1).

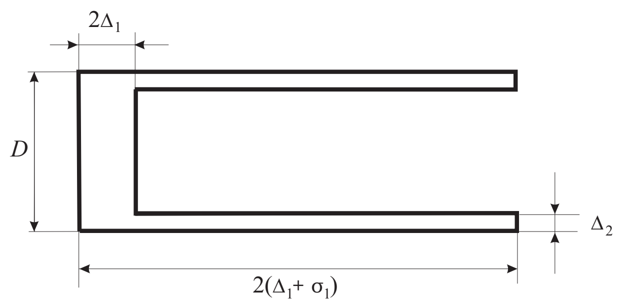

Figure 1: Hollow cylinder ("cartridge")

We calculate the combined body parameters appearing in the inequality (1), i. e., the distance from the center of mass of a body to the front circular butt end σ, and also the (principal) radius of inertia of a body ρ.
Then the left-hand side of (1) under the certain admissions for $h = 0,1$ is reduced to the following equality in critical case:

$$\Delta_1 \left(-\frac{1}{4}\right) + \sigma_1 \Delta_2 \left(\frac{7}{2}\right) - 4\sigma_1^2 \Delta_2 = 0. \tag{2}$$

Let find the critical value σ_1^* of the dimension-free length of the lateral partitions of the combined body. It is equal to

$$\sigma_1^* = \frac{7}{16} + \frac{1}{8}\sqrt{\frac{49}{4} - 4\frac{\Delta_1}{\Delta_2}}. \tag{3}$$

From Eq. (3), we see that the value Δ_1/Δ_2 can oscillate in the following restrictions only:

$$0 < \frac{\Delta_1}{\Delta_2} < \frac{49}{16} = 3,0625. \tag{4}$$

Conclusions

Under the studying of model considered, we find the sufficient conditions of the asymptotic stability of one of the key regime (rectilinear translational deceleration). In application to the homogeneous circular cylinders, we represent the concrete estimations on its inertia-mass characteristics, herewith, we take into account the results of the experiments executing earlier, including the results in obtaining of the dimension-free parameters of the water action on the cylinders.
In this work, we also show that under the certain conditions on the higher derivatives of the medium action functions (the arm of the action force and the coefficient of resistance) it makes possible the presence in the system the stable as well as unstable auto-oscillating regimes of motion. Therefore, the measurement of higher derivatives of these medium action functions is the resistless difficulty, since for every concrete the body, not only the explicit form, but even the signs of higher derivatives are not known in the separate points of such functions.
Under the process of application of the methods (which are obtained earlier) of studying of dissipative dynamic systems of the certain forms arising in the problem on free deceleration, we obtain the new multi-parameter family of the phase patterns on two-dimensional cylinders of the quasi-velocities; such family consists of the infinite set of topologically nonequivalent patterns changing its topological type by the degenerate way under the variation of the system parameters. This family obtained possesses the stable as well as unstable self-oscillatory regimes in the finite range of the angle of attack. Herewith, the domain of the physical parameters is the set of the finite measure in all infinite-measured space of the system parameters, therefore, the patterns obtained are the typical.
The obtained results allow to construct the hollow circular cylinders (i. e., "the cartridge case"), the use of which can provide the necessary stability under the execution of the additional natural experiments [3].

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References

[1] Shamolin M. V. (2007) Methods for Analysis Variable Dissipation Dynamical Systems in Rigid Body Dynamics. Ekzamen, Moscow.
[2] Shamolin M. V. (2011) Rigid body motion in a resisting medium. *Matem. Mod.* **23:12**:79–104.
[3] Shamolin M. V. (2015) Simulation of Rigid Body Motion in a Resisting Medium and Analogies with Vortex Streets. *Matem. Mod.* **27:1**:33–53.