DOI 10.1007/s10958-015-2212-0
Journal of Mathematical Sciences, Vol. 204, No. 6, February, 2015

SESSIONS OF THE WORKSHOP OF THE MATHEMATICS AND MECHANICS DEPARTMENT OF LOMONOSOV MOSCOW STATE UNIVERSITY, "URGENT PROBLEMS OF GEOMETRY AND MECHANICS" NAMED AFTER V. V. TROFIMOV

D. V. Georgievskii and M. V. Shamolin UDC 51(09)

(For the beginning of the list of sessions see *J. Math. Sci.*, **154**, No. 4, 462–495 (2008); **161**, No. 5, 603–614 (2009); **165**, No. 6, 607–615 (2010); **187**, No. 3, 269–271 (2012).)

Session 221 (February 12, 2010)

D. V. Georgievskii.

On possible statements of the dynamical problem in terms of stresses in the isotropic theory of elasticity.

In the case where loadings are given on the whole surface of a deformable rigid body, it is convenient to state and solve (by numerical and analytical methods) the boundary-value problem for determining the stress-strain state in terms of stresses. We discuss some features and the efficiency of four statements of the dynamical problem in the linear isotropic theory of elasticity.

Session 222 (February 19, 2010)

V. I. Gorbachev.

Longitudinal oscillations of rods with variable parameters.

We consider the problem of longitudinal oscillations of a nonhomogeneous rod with variable cross-section (the original problem). The oscillation process is described by a hyperbolic second-order differential equation with coefficients depending on the coordinate. In the case where the coefficients are discontinuous, the equation is understood in the generalized sense. Along with the original problem, we also consider the accompanying problem for a rod with constant parameters (Young's modulus, density, cross-section) and with the same input data as for the original problem. The solution of the accompanying problem is much easier than the solution of the original problem, and in many cases it is possible to obtain its exact analytical solution. For the solution of the original problem, we obtain an integral representation through the solution of the accompanying problem and Green's function of the original problem. If the accompanying problem has a smooth solution, the integral relation implies a representation of the solution of the original problem as a series with respect to the coordinate and time derivatives of the solution of the accompanying problem. Moreover, the coefficients of the derivatives depend only on the coordinates; they are weighted moments of Green's function for one of the two coordinate variables. For these coefficients, we construct a recurrent system of ordinary differential equations. We discuss in detail a special case of the general dynamic problem of determining the eigenfrequencies of longitudinal oscillations of a rod with variable parameters. We obtain exact frequency equations for various options for fixing the ends of the rod.

Translated from Sovremennaya Matematika i Ee Prilozheniya (Contemporary Mathematics and Its Applications), Vol. 88, Geometry and Mechanics, 2013.

Session 223 (February 26, 2010)

T. I. Garyaeva and D. V. Georgievskii.

On the first boundary-value problem of the theory of elasticity for a cylindrical layer with strongly differing characteristic dimensions.

We analyze the principal terms of general asymptotic expansions of solutions to the first boundary-value problem in the three-dimensional elasticity theory for displacements (quasi-statics, compressibility) for a cylindrical layer. The natural small parameter of the problem is the ratio of the thickness of the layer to the length of the generatrix.

Session 224 (March 19, 2010)

I. A. Buyakov.

Features of deformation of a spirally armored shell.

Session 225 (April 2, 2010)

N. A. Belov and V. A. Kadymov.

On a boundary-value problem for a a thin plastic layer.

The report is devoted to the study and solution of Ilyushin's boundary-value problem on spreading of a thin plastic layer between approaching rigid plates.

It is known that a plastic medium in such flows can be adequately described by a model of a viscous liquid. A solution obtained earlier in the approximation of the ideal-fluid model does not satisfy a dynamic condition at the free boundary. However, based on this solution, one can also obtain the evolution equation for the boundary.

This work is devoted to the analytical study of the boundary-value problem. In a neighborhood of the boundary layer, we obtain a solution that satisfies all boundary conditions. If the flow of an ideal fluid is directed along the normal to the boundary, then the velocity of the flow in the boundary layer has nonzero tangent component. We also show that for a sufficiently smooth boundary, the evolution equation for the boundary coincides with an equation obtained earlier.

Session 226 (April 9, 2010)

D. V. Georgievskii.

On the work of the 81st Annual Meeting of the Society for Applied Mathematics and Mechanics, March 20–27, 2010, Karlsruhe, Germany.

Session 227 in the framework of the Conference of young scientists of the Faculty of Mechanics and Mathematics of the M. V. Lomonosov Moscow State University and the International conference of students, postgraduates, and young scientists "Lomonosov–2010" (April 27, 2010).

1. *M. V. Vassilieva.*

Numerical simulation of single-phase filtration on multiprocessor systems.

2. *T. V. Yakovleva.*

Control of complex nonlinear vibrations of flexible beams.

Session 228 (7 May 2010)

V. A. Danilov.

Calculation of time-dependent characteristics of axially symmetric elastoviscoplastic currents.

We propose new formulations and numerical and analytical methods for solving boundary-value problems in the theory of elastoviscoplasticity of the start, acceleration, and braking to complete

stopping of axisymmetric flows with subsequent discharge. We take into account the accumulation of residual stresses and strains. For viscoplastic flows, we find conditions for the appearance and motion of the boundaries of an elastic core. The Taylor–Couette flow between two coaxial cylinders is studied in detail.

Session 229 (4 June 2010)

M. U. Nikabadze.

Some problems of micropolar elasticity theory.

We consider some problems of micropolar elasticity theory. In particular, equations of motion (equilibrium) in stress tensors and moment stresses in the three- and two-dimensional theories are reduced to statements on the absence of volume load and inertial forces. We obtain a formula that expresses the stress tensor through the tensor of moment stresses and introduce tensor-functions of stresses.

The equations of motion in terms of displacements and rotations are written in operator form. For an arbitrary anisotropic material, we introduce four differential tensor-operators and obtain expressions for the corresponding differential tensor-operators for different cases of anisotropy. We discuss isotropic, transversally isotropic, and orthotropic materials. Moreover, we introduce the matrix differential tensor-operator whose differential subtensor-operators are the tensor-operators introduced above; this matrix differential tensor-operator allows one to write the equations of motion in the form of a single matrix differential tensor-operator equation. The case of an isotropic material with a center of symmetry is studied in detail. We obtain expressions for differential tensor-operators of cofactors and determinants of all differential tensor-operators introduced above, except for one tensor-operator whose determinant vanishes. The expressions for the corresponding matrix differential tensor-operator of the cofactors and the determinant are also obtained in the case where the interior tensor of inertia of the material is an arbitrary tensor of rank 2 in principal axes.

Further, we introduce differential tensor-operators and matrix differential tensor-operators, which, if applied to the corresponding equations, allow one to split the system of equations and obtain separate equations for unknown vector-functions (vectors of displacement and rotation). Unlike the Galerkin representation, the boundary conditions of the boundary-value problems are preserved. We also obtain representations of solutions of the Galerkin problem. Problems similar to those described above are also considered for the classical elasticity theory as particular cases.

We present three methods of obtaining formulas of the general complex representation in the planar micropolar elasticity theory taking into account volume loads in nonisotermic processes. In two methods, formulas are represented through two analytic functions of a complex variable and the general solution of the inhomogeneous Helmholtz equation and in the third method through three analytic functions of a complex variable.

Session 230 (18 June 2010)

V. E. Palosh.

Stability of nonconservative mechanical systems.

We examine the stability of equilibria for four nonlinear problems.

First problem: a double pendulum loaded with tracing and conservative forces. The joints of the pendulum possess viscoelastic properties. The stability problem is solved for critical cases of one zero root, two imaginary roots, and one zero and two imaginary roots.

Second problem: a homogeneous rod moving under the action of a constant tracing force applied to one of its ends. The emergence of the destabilization paradox is shown.

Third problem: stabilization of the motion of a dynamically symmetric satellite by using exterior moments. In the linear case, we examine the possibility of the satellite with respect to the center of

mass by using constant moments. In the nonlinear case, we consider the critical case of stability of two pairs of imaginary roots and find the conditions of asymptotic stability.

Fourth problem: a solar sail. In the linear statement, we examine the Lyapunov stability of the equilibrium of a solar sail. In the nonlinear statement, we show that the proof of Lyapunov stability is impossible. We examine the stability of the zero equilibrium for a variety of initial conditions.

Session 231 (September 10, 2010)

D. V. Georgievskii.

Eigenvalue problem for the generalized Orr–Sommerfeld equation in the theory of hydrodynamical stability.

Based on the method of integral relations, we analytically examine the stability of a number of one-dimensional plane-parallel stationary Newtonian flows. The mathematical statement is reduced to eigenvalue problems for the Orr–Sommerfeld equation. As boundary conditions, we consider vanishing of all components of the perturbation of the velocity on both boundaries of the layer (in this case, we have the classical Orr–Sommerfeld problem); vanishing of all components of the perturbation of the velocity on one boundary and vanishing of the perturbation of the tangent component of the stress vector and the normal component of the velocity on the other boundary; vanishing of all components of the perturbation of the velocity on one boundary and the requirement that the other boundary is free.

Boundary condition for the last case contain the spectral parameter. For kinematic conditions, we improve lower estimates of the critical Reynolds number. In other cases, we develop the method of integral relations, which leads to new stability results.

Session 232 (September 17, 2010)

S. V. Bogomolov.

Problem on stochastic diffusion models in gas dynamics.

The accuracy and efficiency of numerical algorithms in gas dynamics can be improved by constructing a hierarchy of mathematical models based on micro-macro representations.

The approach is usually based on the Boltzmann equation whose dimensionless form contains the coefficient 1/Kn of the collision integral, where the parameter Kn (the Knudsen number) depends on the space variable x.

In view of the current high demands placed on the quality of computing technologies, the whole domain in which calculations are performed is partitioned into subdomains with different properties. If Kn is of the order of unity, then it is a subdomain in which the Boltzmann equation is needed. In domains where Kn is moderately small, one can use the Kolmogorov–Fokker–Planck equation whose coefficients are determined by the collision model and can be calculated in explicit form under some simplifying assumptions. This nonlinear equation with respect to the seven-dimensional distribution function in the phase space is simpler than the Boltzmann equation; instead of the collision integral it contains the transfer operator with diffusion in the velocity space, which can be called the model collision integral.

For moderate values of Kn, one can also obtain a macroscopic description: the equations of stochastic quasi-gas dynamics that are related to the Kolmogorov–Fokker–Planck equation by the coefficients obtained by averaging in space and time. For very small Kn these equations are similar to the Navier–Stokes equations.

A micro-macro "bridge" described in the language of deterministic equations can be constructed by using the theory of stochastic processes starting from the system of stochastic differential equations that describe the gas for moderate and small values of the Knudsen number. We obtain a set of stochastic models that generate a sequence of Monte-Carlo methods that are promising from the

point of view of supercalculations. Our approach differs from other methods of construction of quasi-gas-dynamical equations by underlying hypotheses that allow one to obtain a simpler description of a gas as compared to a kinetic description.

Session 233 (October 1, 2010)

K. V. Kvachev.

Lyapunov–Movchan method in a dynamical problem of elasticity theory.

We consider a rectangular plate of constant thickness made of a homogeneous, elastic, isotropic material governed by Hooke's law. Two parallel edges of the plate are pivotally mounted, one edge is fixed, and the fourth edge is free. A supersonic flow of a gas flows around the plate; this flow is parallel to the pivotally mounted edges and is directed from the fixed edge to the free edge. We obtain stability conditions for oscillations of the plate and find the critical speed by using the Lyapunov–Movchan stability theorem.

Session 234: a joint session with the educational and scientific seminar of the department of Applied Mathematics of the Bauman Moscow State Technical University (October 8, 2010)

M. U. Nikabadze.

On the statement and methods of solution of boundary-value problems in the theory of thin bodies.

We consider a parametrization of the domain of a thin body with two small sizes in the case where an arbitrary line is chosen as the base line, and the classical parametrization of the domain of a thin body, i.e., when the median line is taken as the base line. We present three-dimensional statements of problems for a thin elastic body with two small sizes for considered parametrizations of the domain of the body. Based on these statements and using the moment theory with respect to the system of Legendre polynomials, we obtain the corresponding statements of problems in moments. We perform the expansion of mechanical functions with respect to the system of Legendre polynomials in one or in two transversal coordinates. Further, using the method of normalized moments for stress tensors and moment stressed and simplified method of the reduction of an infinite system of equations in moments to a finite system, we obtain the statements of problems of the zero, first, and second approximations. We also establish conditions under which equations of the first approximation imply the equations of the classical theories (Euler–Bernoulli and Timoshenko).

Session 235 (October 15, 2010)

M. V. Shamolin.

Survey of integrable cases in the dynamics of a four-dimensional rigid body in a nonconservative field.

We review complete results on the study of the equations of motion of a dynamically symmetric four-dimensional (4D) rigid body in a nonconservative field that are currently available. The form of these equations is borrowed from the dynamics of realistic two-dimensional (2D) and three-dimensional (3D) rigid bodies interacting with a resisting medium according to jet flow laws such that the body is under the action of a nonconservative force couple that forces the center of mass of the body to move rectilinearly and uniformly.

In the report, we discuss some integrable cases of the problem of motion of a body in a resisting medium that fills four-dimensional space, under the action of a tracing force that allows one to reduce the order of the general system of dynamical equations of motion.

Earlier, the author proved the complete integrability of the equations of plane-parallel motion of a body in a resisting medium under the conditions of jet flow where the system of dynamical equations

possesses a first integral which is a transcendental function (in the sense of complex analysis, i.e., as a function of a complex variable having essential singularities) of quasi-velocities. In those works, it was assumed that the interaction of the body with the medium is concentrated on a part of the surface of the body that has the form of a (one-dimensional) plate. Later, the planar problem was generalized to the spatial (three-dimensional) case, in which the system of dynamical equations also has a complete set of transcendental first integrals. Here we already assume that the interaction of the body with the medium is concentrated on a part of the surface of the body that has the form of a planar (two-dimensional) disk.

In the present work, we generalize some known results concerning integration of the equations of motion of two-dimensional and three-dimensional rigid bodies under the action of a nonconservative moment and examine the equations of motion of a dynamically symmetric four-dimensional rigid body. The structure of these equations is preserved after the generalization to higher-dimensional cases.

Session 236 (November 12, 2010)

B. N. Khimchenko.

Hamiltonian systems and formula for calculation of the Laplacian of the eikonal.

The necessity of the calculation of the Laplacian of the eikonal appears in a series of problems of mathematical physics, for example, in transport equations. Integration of such equations leads to the construction of asymptotic expansions of solutions of differential equation with respect to a small parameter.

We show that the conventional formula for calculation of the Laplacian of the eikonal is invalid and propose a new scheme for calculation using the matrix Riccati equation.

Session 237 (November 19, 2010)

I. A. Buyakov and G. E. Tashchilova.

On the carrying capacity of compressed anisotropic tubular rods.

Using the momentless theory of anisotropic shells, we study the carrying capacity of tubular anisotropic rods under axial compression. We obtain the dependence of the carrying capacity of tubular rods on the complex of elastic characteristics of anisotropic material.

Session 238 (December 3, 2010)

N. N. Shamarov.

Maslov–Poisson measure method.

We present a method of solution of evolutionary equations whose particular case is equivalent to Maslov's method for the solution of a certain class of Schrödinger equations by using the functional integral over a complex-valued countably additive measure of Poisson type. This measure is defined on the space of trajectories in the dual space of physical coordinates and the corresponding integral can be called the integral by trajectories in the momentum space.

While the original Maslov's method is based on an exponential series of Dyson type, the method discussed is based on product formulas of Chernov and Trotter type for approximations of one-parameter operator semigroups. This method is applicable to equations with matrix coefficients (in general, not commuting), for example, the Schrödinger equation with matrix effective potential, the famous Dirac equation for a relativistic electron, and the classical heat equation with matrix effective source (sink) of heat. Moreover, the method can be adapted to analogs of the heat equation in which the "space variable" runs through a space over the field of -adic numbers (not necessarily finite-dimensional), and the role of the Laplace operator is played by the Vladimirov operator (or its infinite-dimensional analog).

The modified method also leads to integrals by means of trajectories different from Feynman integrals in the phase space for a solution of the classical Schrödinger equation.

Session 239 (December 10, 2010)

A. V. Mokeev.

On some problems of differential diagnostics.

We discuss and solve two basic problems of differential diagnostics: the monitoring problem and the problem of fault diagnosis. The solutions of these problems are based on the mathematical model of the motion of the object considered (in terms of dynamical control systems), including the domain of its initial conditions, the a priori list of possible faults, and the mathematical model of the motion of the object under a corresponding fault. We define the notion of a neighborhood of a reference fault and, based on it, introduce a topology on the space of reference faults (the so-called diagnosis space). The monitoring problem is solved by the construction of a monitoring surface; it can be solved in both deterministic or statistical statements. We propose a method of construction of the monitoring surface by using the Monte-Carlo method. For the problem of fault diagnosis, we prove a diagnosis theorem and, as a consequence, propose two diagnosis algorithms.

Session 240 (December 24, 2010)

S. A. Dovbysh.

Foundations of Ziglin's theory (nonintegrability of dynamical systems and monodromy groups of equations in variations).

This report is an introductory lecture for untrained readers.

We discuss principal ideas and results of the theory developed by Ziglin, which states sufficient conditions of nonexistence of additional first integrals of dynamical systems in terms of properties of the normal equation in variations (NEV) along a known particular solution of the initial system (IS).

We consider the following notions and results: construction of a homogeneous polynomial (respectively, rational) first integral of the NEV from an analytic (respectively, meromorphic) first integral of the IS; a lemma that states that the existence of k functionally independent meromorphic first integrals of the IS implies the existence of k functionally (and, equivalently, algebraically) independent rational first integrals of the NEV; the monodromy group of the NEV and the relation between the integrability of the IS and the existence of rational first integrals of the monodromy group; necessary conditions of complete integrability of the Hamiltonian IS in terms of commutator properties of two elements of the monodromy group, one of which is nonresonant; a version of this result for systems with two degrees of freedom and its generalization to the case of a nondiagonalizable element.

Session 241 in the framework of the International Scientific Symposium "Problems in Mechanics of Solids" devoted to the 100th Anniversary of A. A. Il'yushin (January 21, 2011) (January 21, 2011)

D. V. Georgievskii and R. Wille.

Asymptotical integration in boundary-value problems on a perfect rigid plastic flow in a thin layer.

The method of asymptotic integration for a series of boundary-value problems on incompressible, perfect, rigid, plastic flow in a thin plane layer under loading is developed by analytic methods. The material of the layer may occupy an arbitrary domain. We propose an algorithm for the construction of an asymptotic solution and consider the possibility of a perfect, rigid, plastic flow along one of the coordinate lines. The results for some particular cases are discussed: the classic Prandtl problem and its axially symmetric analogs.

Session 242 (February 11, 2011)

L. E. Evtushik.

The Ostrogradsky theorem on the Hamiltonization of the Euler–Lagrange equations for Lagrangians with higher-order derivatives.

Session 243 (February 18, 2011)

V. I. Van'ko.

Cylindrical shell under an exterior pressure: nonclassical solution of the problem on large displacements.

Session 244 (February 25, 2011)

I. A. Buyakov.

The sudden stratification and bulging of a three-layer rod under compression.

We consider the problem of the exhaustion of the bearing capacity of a three-layer rod under compression due to the sudden detachment of its supporting layers from the filler. We find the characteristic length of the lifting zone which must be taken into account in the assignment of additional transversal constraints between the layers under insufficient adhesive or cohesive solidity of the adhesive bond.

Session 245 (March 11, 2011)

D. V. Georgievskii.

Gravitational stability of some two-layer vertically moving systems.

We study the evolution of small initial perturbations in a system consisting of a heavy layer of a Newtonian fluid that covers the half-space of an ideal fluid with another density. This system as a rigid body can move in the vertical direction by a given law. Linearizing the equations and boundary conditions, we obtain the characteristic equation and consider the large-viscosity limit.

Session 246 (March 18, 2011)

E. S. Perelygina.

Longitudinal bending of an elastic-plastic rod under free diagram σ-ε.

Session 247 (March 25, 2011)

S. A. Dovbysh.

Nonintegrability of dynamical systems and monodromy groups and Galois groups of linear ordinary differential equation. Some aspects of Ziglin's theory (continuation).

Session 248: a joint session with the educational and scientific seminar of the department of Applied Mathematics of the Bauman Moscow State Technical University (April 8, 2011)

D. V. Georgievskii.

Asymptotic analysis of the Prandtl problem in the dynamical statement.

The dynamical statement of the problem of compression of a thin, ideal, rigid plastic layer by absolutely rigid plates that move towards each other with constant speeds contains two specific dimensionless parameters. One of them, a small geometric parameter α equal to the ratio of the thickness of the layer to its length explicitly depends on time; moreover, its order of smallness increases with time. The other dimensionless parameter, the reciprocal Euler number, is independent of time; we assume that it is much less than unity. Depending on the relation between these parameters, i.e., in

different time intervals, we construct solutions in the form of expansions by integer powers of α using the procedure of asymptotic integration.

Session 249 (April 15, 2011)

M. M. Kantor.

Modeling of deformation of thin bodies with two small sizes.

We consider a parametrization of the domain of a thin body with two small sizes in the case where an arbitrary line is chosen as the base line, and the classical parametrization of the domain of a thin body, i.e., when the median line is taken as the base line. We present three-dimensional statements of problems for a thin elastic body with two small sizes for these parametrizations of the domain of the body. We develop the theory of moments with respect to Legendre polynomials that allows one to calculate the moment of order (m, n) of any expression. Based on these statements and using the theory of moments with respect to the system of Legendre polynomials, we obtain the corresponding statements of the problem in moments. We perform an expansion of mechanical functions with respect to the system of Legendre polynomials in one or in two transversal coordinates. We construct various approximations from zeroth to fifth order for the classical and micropolar theories. To satisfy the boundary conditions on the front surfaces, we use the method of normalized moments of the stress tensors and moment stresses and the method of correcting terms.

We write programs that allow one to perform a numerical simulation of test problems and compare the obtained numerical results with classical solutions, including the solution obtained by the method of finite elements. The comparison is performed for the problem for a micropolar two-dimensional domain, the problem of the action of concentrated forces on a two-dimensional domain, and the problem for a two-layer two-dimensional domain.

Session 250: anniversary session (April 29, 2011).

Session 251 in the framework of the XV International Conference "Dynamical Systems: Modelling and Stability" (May 27, 2011).

1. *S. A. Agafonov and I. A. Kostyushko.*

On the reduction of a nonautonomous linear system and an application.

We consider a nonautonomous linear system of ordinary differential equations. We reduce it to an autonomous system and then study it by well-known methods.

2. *V. I. Van'ko.*

Cylindrical shell under an exterior pressure: nonclassical solution of the problem on large displacements.

In previous works, the author considered the problem on large displacements of points of the median surface of an infinitely long, circular, cylindrical shell under the action of an exterior hydrostatic pressure. The author has succeeded in following the deformation process until to complete flattening by using a kinematic scheme. In this work, the influence of the parameters of characteristic length and thickness and the boundary-value condition on the deformation process is examined.

3. *D. V. Georgievskii.*

On generalized Orr–Sommerfeld problems in continuum mechanics.

We study the generalized Orr–Sommerfeld equation in the linearized theory of hydrodynamical stability.

Methods of integral relations that have been intensively developed in the past decades for materials with complicated defining relation are also effective for this spectral problem. These methods allow one to complete sufficient estimates of the stability of a process without knowing any exact or approximate

solution of the linearized problem at each time instant. Generalizations of the Squire theorem for spectral problems are also obtained.

4. *M. V. Shamolin.*

Comparison of completely integrable cases in the dynamics of 2D, 3D, and 4D rigid bodies in nonconservative fields.

The complete integrability of the equations of motion of a four-dimensional rigid body has been examined in a large number of works. The author has succeeded in generalizing the equations of motion of low-dimensional (two- and three-dimensional) rigid bodies in nonconservative fields to the motion of a four-dimensional rigid body in a similar field.

As a result, the author obtained several new integrable cases in the problem of motion of a body in a resisting medium that fills four-dimensional space under the action of some tracing force that allows one to reduce the order of the general system of dynamical equations of motion.

In the present work, we generalize some previously known results on integration of the equations of motion of two- and three-dimensional rigid bodies under the action of a nonconservative force moment and study the equations of motion of a dynamically symmetric four-dimensional rigid body in one of the two possible cases, depending on the principal moments of inertia. The structure of such equations of motion is preserved in some sense when they are generalized to higher-dimensional cases.

Session 252: Towards the XX All-Russia Congress on Fundamental Problems of Theoretical and Applied Mechanics (June 10, 2011).

Session 253 (September 9, 2011)

V. S. Yushutin.

Stability of a deformable channel in the case where a nonlinearly viscous medium with power hardening law flows in it.

We consider a dynamical model of a system consisting of a cylindrical deformable vessel and a nonlinearly viscous medium with power hardening law that flows in the vessel. The flow and the deformation are assumed to be axisymmetric. We examine the stationary solutions and their stability with respect to small perturbations. In the space of dimensionless parameters of the problem, we construct stability domains.

Session 254 (September 16, 2011)

O. A. Ryabova.

Mathematical modeling of stress-strain states of bodies with rigid circular inclusions under finite planar deformations.

We develop a method of construction of mathematical models that under finite planar deformations, describe the stress-stain states of infinite nonlinearly elastic and viscoelastic bodies with rigid circular inclusions that appear after loading.

Session 255 (September 30, 2011)

K. V. Kvachev.

Statement of the dynamical problem on the stability of oscillations of a cylindrical shell in a supersonic gas flow.

We consider a cylindrical shell of constant thickness made of a homogeneous, elastic, isotropic material governed by Hooke's law. One edge of the shell is fixed and the other is free. A supersonic flow of a gas streamlines the plate; this flow is parallel to the pivotally mounted edges and is directed from the fixed edge to the free edge. We present the statement of the problem under the assumption that deformations are small and the "piston" theory is applicable.

Session 256 (October 14, 2011)
I. A. Buyakov.
Generalized edge effect in thin-wall laminate bodies.

Session 257 (October 21, 2011)
V. M. Ovsyannikov.
Using higher-order terms of the finite-difference continuity Euler equation in phenomena of mixing and generation of sound.

Session 258 (October 28, 2011)
D. V. Kuznetsova and I. N. Sibgatullin.
Intermittency on the background of quasi-periodic regimes in penetrative convection.

We consider convection in a planar layer of water in a temperature interval that includes the maximum density point 4° C. We describe qualitative features of periodic, two-periodic, and quasi-periodic regimes. We examine the stability of stationary and periodic regimes on large horizontal scales. When the stability of quasi-periodic regimes is lost, intermittency occurs.

Session 259 (November 11, 2011)
M. V. Shamolin.
Systems with variable dissipation: approaches, methods, and applications.

This work is a survey of integrable cases in the dynamics of two-, three-, and four-dimensional rigid bodies in nonconservative fields. The problems are described by dynamical systems with variable dissipation with zero mean.

The problem of the search for complete sets of transcendental first integrals of systems with dissipation is quite urgent, and many papers are devoted to it. We consider a new class of dynamical systems that have a periodic coordinate. Owing to the fact that such systems have nontrivial symmetry groups, we prove that such systems possess variable dissipation, which means that on the average for a period with respect to the periodic coordinate, the dissipation in the system is equal to zero, although in various domains of the phase space either energy pumping or dissipation can occur. Dynamical systems that appear in the dynamics of a rigid body are analyzed and several new cases of complete integrability of the equations of motions in transcendental functions that can be expressed through finite combinations of elementary functions are detected. Some generalizations of integrability conditions for more general classes of nonconservative dynamical systems in the dynamics of a four-dimensional rigid body are obtained.

Session 260 (November 18, 2011)
I. Kh. Sabitov.
On an ill-posed boundary-value problem of Markushevich.

Session 261 dedicated to the memory E. V. Lobanov (November 25, 2011)
D. V. Georgievskii.
Asymptotics in the Prandtl problem in a dynamical statement.

Session 262 (December 2, 2011)

M. U. Nikabadze.

A complete system of eigentensors for a positive-definite symmetric tensor of any even rank.

Session 263 (December 16, 2011)

A. R. Ulukhanyan.

On the general dispersion equation for a homogeneous anisotropic micropolar medium.

We obtain a general dispersion equation for a homogeneous, anisotropic micropolar medium. We consider a homogeneous, micropolar, infinite medium possessing a center of symmetry. The equations of motion of the micropolar theory and the defining relations of the micropolar medium are written in components. Substituting the defining relations in the equations of motion, we obtain equations for the homogeneous, anisotropic micropolar medium in terms of components of translation and rotation vectors. Since the equations obtained are hyperbolic, we find solutions in the form of two wave functions. As usual, equating the determinant of the system obtained to zero, we derive the general dispersion equation, which allows one to find the speed of waves in the infinite, micropolar anisotropic medium. This also shows that in each direction, no more than six waves can propagate; to find them, one needs to solve biquadratic equations. For a micropolar, isotropic, elastic medium, dispersion equations were obtained, for example, by V. Novatsky. M. Nikabadze proved that the mechanical properties of micropolar, elastic, transversally isotropic media and orthotropic media are described by 20 and 30 independent components, respectively. Knowing the number of independent components, we can obtain similar equations for transversally isotropic and orthotropic media from the general dispersion equation.

We present general representations of solutions of hyperbolic equations of fourth and sixth orders. Earlier, the author obtained a system of equations of motion for a micropolar, anisotropic, elastic medium of variable thickness with respect to the system of orthogonal Legendre polynomials. Based on this system, in the case of the classical theory, for isotropic and transversally isotropic prismatic bodies of constant thickness, hyperbolic equations of fourth and sixth orders in the first and second approximation, respectively, were obtained. By the Fourier method of separation of variables, hyperbolic equations of fourth and sixth orders are reduced to elliptic equation of the same orders. By the Vekua method of representation of general solutions of elliptic equations of order $2n$ by n analytic functions, we present general representations of solutions of hyperbolic equations of fourth and sixth orders. We also consider the problem on the bending of a rectangular plate in the first approximation.

Session 264 (March 2, 2012)

V. Yu. Alekseev.

Some methods of solution of dynamical problems in the mechanics of composites with periodic structure.

Session 265 (March 23, 2012)

D. V. Georgievskii.

Adjusting experiments for finding material functions for tensor nonlinear defining relations.

We describe the scheme of an adjusting experiment for finding material functions of two invariants that enter in the defining relations of a tensor nonlinear incompressible medium. As a basic flow of such medium, we take a combination of radial spreading and two one-dimensional shifts in two mutually perpendicular directions that are realized in a cylindrical layer. At the boundaries of the

cylinders, leaking can occur, but the tangential component of the velocity of the medium is equal to the velocity of the motion of the cylinders. We state the problem of the existence of a viscous potential for specific media.

Session 266 (March 30, 2012)

M. V. Shamolin.

On the problem of a pendulum in a nonconservative case.

This work is a survey of integrable cases for the equations of motion of a fixed pendulum in a nonconservative field. These problems are described by dynamical systems with variable dissipation with zero mean.

The problem of the search for a complete sets of transcendental first integrals of systems with dissipation is quite current and many papers are devoted to it. We consider a new class of dynamical systems that have a periodic coordinate. Owing to the fact that such systems have nontrivial symmetry groups, we prove that the systems possess variable dissipation, which means that on the average for a period with respect to the periodic coordinate, the dissipation in the system is equal to zero, although in various domains of the phase space, either energy pumping from outside or dissipation can occur. Dynamical systems that appear in the dynamics of a rigid body are analyzed and several new cases of complete integrability of the equations of motions in terms of transcendental functions that can be expressed through finite combinations of elementary functions are analyzed. Some generalizations of integrability conditions for more general classes of nonconservative dynamical systems in the dynamics of a four-dimensional rigid body are obtained.

Session 267 (April 6, 2012)

V. V. Vedeneev.

Panel flutters under low supersonic speeds.

In the theory of panel flutters (instabilities of elastic plates in a gas flow), two types of stability violation are known: flutter of fixed type and one-mode flutter. The first type appears in the case of large supersonic speeds of the flow; it is studied in detail by the "piston theory," i.e., an approximate expression for the perturbation of the pressure acting on the plate for large Mach numbers . The agreement of the boundaries of the flutter of fixed type with experiments for $M > 1.7$ is satisfactory.

The one-mode flutter appears for $1 < M < 2$ and have not yet been studied since the exact theory of potential gas flows that is necessary for its study leads to complicated integro-differential equations. Moreover, in experiments, the one-mode flutter has not been occurred.

We present the results of recent theoretical and experimental studies of plate flutter for low supersonic speeds. Using the asymptotic global-instability method, we solve the problem on the flutter of a plate of large but finite size. We obtain criteria of fixed and one-mode flutters and examine the structure of the spectrum of the system. We also study the physical reasons of the amplification of oscillations under one-mode flutter.

We study the influence of the boundary layer on one-mode flutter. The stabilization or destabilization of perturbations of the plates by a boundary layer is determined by the profile of the layer and its thickness. We find that amplification of oscillations of the plate for a specific shape of its profile and thickness of the boundary layer is possible.

We numerically solve the problem of stability of a plate in a potential flow for arbitrary sizes of the plate. The boundary of the flutter (both one-mode and fixed) are compared with asymptotic results.

We perform an experimental study of panel flutter in the transonic range of speeds. The appearance of one-mode flutter in a realistic construction is identified for the first time.

We theoretically examine nonlinear oscillations of the plate and the growth of the amplitude of a limit cycle of oscillations. We prove the possibility of simultaneous existence of different limit cycles.

Session 268 (April 13, 2012)

V. I. Van'ko.

On the stability of elements of constructions.

We study the behavior of the Shanley rod model and present principal statements and features of the longitudinal bending of elasto-plastic elements.

Using a numerical solution of the problem of bending of a rod made of an elasto-plastic material (including accounting of creep) we conclude that the geometrically linear statement of the problem is correct (i.e., to a small increment of the force or time there corresponds a positive increment of the deflection) until the longitudinal force exceeds the bending stiffness of the most highly stressed (by the value of the intrinsic moment) cross-section (in dimensionless values).

We present a nonclassical approach to the study of large displacements of points of the median surface of circular cylindrical shells (infinitely long or of finite length) under an exterior hydrostatic pressure. We consider shells made of linearly or nonlinearly elastic and linearly or nonlinearly viscous (steady creep) materials. We also present examples of numerical solutions of the problems considered comparing them to the results of asymptotic analysis.

In the study of the behavior of an aerodynamical profile in an air flow (plane-parallel motion with three degrees of freedom), we obtain a sufficient condition of instability of equilibria in the Lyapunov sense. We give a generalization of this condition to the system of circular profiles and discuss possible applications of these results to the structural design of split-phase wires of high voltage power lines.

Session 269 (April 20, 2012)

D. V. Georgievskii.

General solutions of systems in stresses that are not equivalent to the classical system of elasticity theory.

We analyze general solutions of some weakened systems of equations in stresses in the isotropic theory of elasticity. These nonequivalent classical systems, in addition to equilibria equations, contain only three of six compatibility equations, diagonal or off-diagonal. We discuss the equivalence of the statements of quasi-static boundary-value problems in terms of stresses of elasticity theory.

Session 270 (April 27, 2012)

M. U. Nikabadze.

Anisotropy in the linear micropolar elasticity theory.

In the linear micropolar elasticity theory of anisotropic bodies without centers of symmetry in the sense of elastic properties, we introduce tensor-columns of the stress tensors and moment stress tensors and also the deformation and bending-torsion tensors. We also introduce the block-tensor matrix of tensors of elasticity moduli; it consists of four tensors of fourth rank. We give representations of the elastic deformation energy and defining relations (Hooke's law) using the introduced tensor-columns and the block-tensor matrix. We define positive-definite block-tensor matrices and prove the positive definiteness of the matrix of the tensor of elasticity moduli. We introduce the notions of an eigenvalue and an eigentensor-column of a block-tensor matrix and consider the problem of finding the eigenvalues and eigentensor-columns.

In explicit form, we construct a complete system of eigentensors for a positive-definite symmetric tensor of any even rank and a complete system of eigentensor columns for a positive-definite symmetric block-tensor matrix consisting of tensors of the same even rank in a space of arbitrary finite dimension. Special cases are considered.

Session 271 (May 11, 2012)

K. V. Kvachev.

The Lyapunov–Movchan method in a problem on the stability of oscillations of a thin elastic cylindrical shell in a gas flow.

By using the Lyapunov–Movchan method, we consider the dynamical stability of an elastic cylindrical shell of finite length that is rigidly clamped on one edge and is free on the other in a supersonic gas flow. Displacements of the shell are subordinated to some restrictions that allow one to integrate one of the equations of motion and construct the Lyapunov–Movchan functional. Applying the Lyapunov stability theorem, we obtain restrictions for the speed and mechanical and geometrical parameters of the system. The results obtained are compared with the results of the problem of stability of axisymmetric oscillations.

Session 272 (May 18, 2012)

V. S. Yushutin.

Integral statement of viscoelastic flows in a cylinder.

We consider a longitudinal axisymmetric flow of a viscoplastic Shvedov–Bingham medium inside a cylinder. The motion is caused by a pressure drop varying in time. Such problems were studied earlier by numerical simulation. The problem contains two variables: the transversal coordinate and time. We propose to apply the semi-inverse method starting from the profile of the longitudinal velocity and integrate the equation of motion with respect to cross-section. Then the problem will depend only on time and the corresponding equations become ordinary differential equations. In the framework of this integral statement, the problem can be considered as a dynamical system with two variables: the velocity and the size of the stiffness core located at the channel axis. The stationary position of the dynamical system corresponds to stationary Poiseuille flows and is a stable-node type point. We construct numerical solutions of the acceleration of the medium by various regimes of the pressure drop. If a pressure drop is absent, then the Shvedov–Bingham medium moves by inertia and, as is well known, stops in finite time. Based on the integral statement of the problem, we obtain estimates of the stop time and compare them with known upper estimates of the stop time in the exact statement of the problem.

Session 273 (June 1, 2012)

V. A. Kadymov and N. A. Belov.

On the spreading of a plastic layer consisting of various media between approaching rigid plates.

Back in the mid-19th century, A. A. Il'yushin proposed an effective two-dimensional, averaged (with respect to the thickness of the layer) mathematical model of a flow of a thin plastic layer between approaching rigid plates. In the framework of this model, he stated the boundary-value problem for a linearly viscous fluid with respect to three unknown functions, two components of the velocity of the flow and the contact pressure in a domain with moving boundary. He also indicated the possibility of simplifying the statement of the problem to a flow of an "ideal fluid." Subsequently, all researchers restricted themselves to this simplified statement.

In the present work, we consider flows of a plastic layer that cannot be described in the framework of the model of an ideal fluid, in particular, the spreading of a layer consisting of various media. For the correct description of such flows, one must use a model of a viscous fluid or the simpler theory of boundary layers.

Session 274 (October 19, 2012)

D. V. Georgievskii and M. V. Shamolin.

Urgent problems of geometry and mechanics: foundations, problems, methods, and applications.

At the session of the seminar, a number of problems were suggested to undergraduate students, graduate students, and researchers. The following domains of research were proposed: relative structural stability in nonconservative dynamical systems; problems on the existence of closed orbits on manifolds of various topological types; the dynamics of a rigid body interacting with a resisting medium under quasi-stationary conditions; integrability and nonintegrability in the classical dynamics and dynamics of strongly nonconservative systems; problems of structural optimization; dynamics of multidimensional rigid bodies; characteristic classes (invariants) of Maslov–Arnold–Trofimov and generalized characteristic classes for dissipative systems; stability of processes in the mechanics of deformable rigid bodies with respect to some classes of perturbations (general methods, approaches, and criteria); stability of flows of materials with complicated rheology and intrinsic structure (composites); stability of rods, plates, shells, and other typical elements of engineering constructions; visco-elastic-plastic flows (stationary and nonstationary) in nonclassical domains; theory of defining relations in biomechanics; the sensitivity of solutions of problems with respect to perturbations of material functions; numerical modeling of the violation of stability and possible physical interpretations.

Session 275 (October 26, 2012)

K. V. Kvachev.

The Lyapunov–Movchan method in dynamical problems of elastic stability.

This report is a presentation of a Ph.D. thesis. The thesis consists of four chapters. The first chapter is a survey of the Lyapunov–Movchan method. The other three chapters are devoted to the mathematical development of this method and applications to problems of aeroelastic stability. In the second chapter, problems on the stability of oscillations of plates of various configurations in a supersonic gas flow are considered. The third and fourth chapters are devoted to problems on the stability of oscillations of cylindrical shells in a supersonic gas flow. For each problem considered, we obtain sufficient stability conditions in terms of an upper estimate of the critical speed. In problems on the stability of oscillations of cylindrical shells, additional restrictions on the change of mechanical and geometrical parameters of the system appear.

Session 276 (November 9, 2012)

R. R. Gadelev.

On the application of concentration tensors for an inhomogeneous half-space.

Session 277 (November 16, 2012)

E. S. Perelygina.

Notes on elasto-plastic bending.

Session 278 (November 23, 2012)

L. E. Evtushik.

On the Hamiltonization of Euler–Lagrange equations with higher derivatives.

Session 279: a joint session with the seminar "Qualitative theory of differential equations" (November 30, 2012)

A. F. Pronevich.

Integrals of systems of exact differential equations.

Session 280 (December 14, 2012)

M. V. Shamolin.

Integrable cases in the dynamics of a multi-dimensional rigid body in a nonconservative field.

This work is a survey of integrable cases in the dynamics of two-, three-, and four-dimensional rigid bodies in nonconservative fields. These problems are described by dynamical systems with variable dissipation with zero mean. We study nonconservative systems for which the usual methods of studying Hamiltonian systems are not applicable. Thus, for such systems, we must "directly" integrate the main equation of dynamics. We propose a more universal exposition of new and previously obtained cases of complete integrability of the equations of motions in terms of transcendental functions in the dynamics of two-, three-, and four-dimensional rigid bodies in nonconservative fields. Of course, in the general case, the construction of a theory of integration of nonconservative systems (even of low dimension) is a quite difficult task. In a number of cases, where the systems considered have additional symmetries, we succeed in finding first integrals through finite combinations of elementary functions. We obtain a series of complete integrable nonconservative dynamical systems with nontrivial symmetries. Moreover, in almost all cases, all first integrals are expressed through finite combinations of elementary functions; these first integrals are transcendental functions of their variables. In this case, transcendence is understood in the sense of complex analysis, when the analytic continuation of a function into the complex plane has essentially singular points. This fact is caused by the existence of attracting and repelling limit sets in the system (for example, attracting and repelling focuses).

Session 281 (December 21, 2012)

D. V. Georgievskii.

Symmetrization of the tensor operator of compatibility equations in stresses in the anisotropic elasticity theory.

We find the general form of the term that symmetrizes the differential rank-4 tensor-operator of the compatibility equations in stresses in the anisotropic elasticity theory. For an arbitrary anisotropy, it includes two arbitrary parameters with the dimension of elastic compliances. The symmetrized compatibility equations themselves contain only one of these parameters.

Session 282 (December 28, 2012)

M. V. Smolentsev.

On the spectra of frequencies of zeros for solutions of third-order linear differential equations.

D. V. Georgievskii
Institute of Mechanics of the M. V. Lomonosov Moscow State University, Moscow, Russia
E-mail: georgiev@mech.math.msu.su
M. V. Shamolin
Institute of Mechanics of the M. V. Lomonosov Moscow State University, Moscow, Russia
E-mail: shamolin@imec.msu.ru