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On stability of certain key types of rigid body motion in a nonconservative field

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In this activity the qualitative analysis of spatial problems of the real rigid body motions in a resistant medium is fulfilled. A nonlinear model that describes the interaction of a rigid body with a medium and takes into account (based on experimental data on the motion of circular cylinders in water) the dependence of the arm of the force on the normalized angular velocity of the body and the dependence of the moment of the force on the angle of attack is constructed. An analysis of plane and spatial models (in the presence or absence of an additional tracking force) leads to sufficient stability conditions for translational motion, as one of the key types of motions. Either stable or unstable self-oscillation can be observed under certain conditions.

1 Introduction

Experimental data on the motion of homogeneous circular cylinders in water [1] show that the dependence of the moment of force of the medium on the angular velocity of the body should also be taken into account.

The nonlinear analysis of the motion of a body with finite angles of attack concentrates on establishing the conditions under which there exist finite-amplitude oscillations near the unperturbed motion, which confirms the necessity of a comprehensive nonlinear analysis. Studies of the plane and spatial models describing the interaction of a rigid body and a medium (in the presence or absence of an additional follower force) have established sufficient stability conditions for translational motion, which is a key type of motion.

Of practical importance is the stability analysis of the so-called unperturbed (translational) motion such that the velocities of points of the body are perpendicular to the plate (cavitator). The present paper is the next stage in the study of a moving rigid body interacting with the medium only by the flat front area (plate). The force exerted by the medium is found using the properties of a quasistationary jet flow [2].

2 Spatial Motion of an Axially Symmetric Rigid Body in a Resisting Medium

Consider a moving homogeneous body of mass m. A portion of its surface is a flat disk. A jet flow is past the body [1, 2]. The other portion of the body's surface is inside the volume bounded by the jet stalling at the disk edge and is not affected by the medium. Conditions are similar when homogeneous circular cylinders enter water. Assume that there are no tangential forces. Then the force **S** applied by the medium to the body at the point N does not change the orientation relative to the body (is normal to the disk) and is quadratic with respect to the velocity of its center D.

If the above conditions are satisfied, the motions of the body include translational deceleration similar to the case of plane-parallel (unperturbed) motion: the body can undergo translational motion along its axis of symmetry, i.e., perpendicularly to the disk plane. We choose the right-hand coordinate system $Dxyz$ with the Dx-axis aligned with the axis of geometrical symmetry of the body and the $Dy-$ and $Dz-$axes fixed to the disk. The components of the angular velocity vector Ω in the system $Dxyz$ are denoted by $\{\Omega_x, \Omega_y, \Omega_z\}$. The inertia tensor of the dynamically symmetric body is diagonalized in the body axes $Dxyz$: diag$\{I_1, I_2, I_2\}$.

We will also use the quasi-stationarity hypothesis and assume for simplicity that $R = DN$ is defined at least by the attack angle α between the velocity vector **v** of the center D of the disk and the straight line Dx. Thus, $DN = R(\alpha, \ldots)$. Moreover, we assume that $S = |\mathbf{S}| = s_1(\alpha)v^2,\ v = |\mathbf{v}|$. For convenience, we introduce (as in the case of plane-parallel motion) an auxiliary alternating function $s(\alpha)$: $s_1 = s_1(\alpha) = s(\alpha)\mathrm{sgn}\cos\alpha > 0$ instead of the coefficient $s_1(\alpha)$. Thus, the pair of functions $R(\alpha, \ldots)$ and $s(\alpha)$ defines the forces and moments exerted by the medium on the disk under such assumptions.

2.1 Dynamic Part of the Equations of Spatial Motion

Let us use the spherical coordinates (v, α, β_1) of the tip of the velocity vector $\mathbf{v} = \mathbf{v}_D$ of the point D relative to the flow to measure the angle β_1 in the plane of the disk. Expressing the quantities (v, α, β_1), using nonintegrable relations, in terms of the cyclic kinematic variables and velocities and supplementing them with the projections $(\Omega_x, \Omega_y, \Omega_z)$ of the angular velocity onto the body axes, we consider them as quasivelocities. Using the theorems on the motion of the center of mass (in

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the body-fixed frame of reference $Dxyz$) and variation in the angular momentum in the same frame, we obtain the dynamic part of the differential equations of motion in the six-dimensional phase space of quasivelocities ($\sigma = DC$). The first group of equations describes the motion of the center of mass, while the second group the motion around the center of mass [1].

Let us consider the class of problems where a rigid body moves through a medium under a follower force acting along the axis of geometrical symmetry of the body and producing classes of motions (imposed constraints) of interest, this force being the reaction of the constraints imposed. Here, the follower force is such that condition $v = \text{const}$ is satisfied all the time. The Routh-cyclic invariant relation $\Omega_x \equiv \Omega_{x0} = \text{const}$ holds at all instants of time. In what follows, we will examine the case where the rigid body does not rotate about its longitudinal axis, i.e., $\Omega_{x0} = 0$. Then the independent dynamic part of the equations of motion in the four-dimensional phase space is given by

$$\begin{aligned}&\dot{\alpha}\cos\alpha\cos\beta_1 - \dot{\beta}_1 v\sin\alpha\sin\beta_1 + \Omega_z v\cos\alpha - \sigma\dot{\Omega}_z = 0,\\ &\dot{\alpha}\cos\alpha\sin\beta_1 + \dot{\beta}_1 v\sin\alpha\cos\beta_1 - \Omega_y v\cos\alpha + \sigma\dot{\Omega}_y = 0,\ I_2\dot{\Omega}_y = -z_N s(\alpha)v^2,\ I_2\dot{\Omega}_z = y_N s(\alpha)v^2,\end{aligned} \tag{1}$$

where y_N and z_N are Cartesian coordinates, in the plane of the disk, of the point N of application of the resisting force. System (1) includes the influence functions y_N, z_N, and s. To determine them qualitatively (by analogy with the case of plane-parallel motion), we will use experimental data on the properties of jet flow.

For simplicity, we will analyze system (1) for the following influence functions; such an analysis can be performed for an arbitrary pair of functions y_N, z_N, and s: $y_N = A\sin\alpha\cos\beta_1 - h\Omega_z/v,\ z_N = A\sin\alpha\sin\beta_1 + h\Omega_y/v,\ s(\alpha) = B\cos\alpha,\ A, B, h > 0$.

The resultant system will be called a reference one. The coefficient h appears in the terms proportional to the rotary derivatives of the moment of hydroaerodynamic forces (drag) with respect to the components of the angular velocity of the body.

System (1) is a dynamic system with variable dissipation and with zero mean (with respect to the angle of attack) [3]. This means that the integral of the divergence of its right-hand side over the period of the angle of attack, which describes the variation in the phase volume (after the appropriate reduction of the system), is equal to zero. The system is semiconservative.

Projecting the angular velocities onto the moving axes not fixed to the body so that $z_1 = \Omega_y\cos\beta_1 + \Omega_z\sin\beta_1,\ z_2 = -\Omega_y\sin\beta_1 + \Omega_z\cos\beta_1$ and introducing dimensionless variables $w_k,\ k = 1, 2$, and parameters by the formulas $h_1 = hB,\ \sigma h_1/I_2 = H_1,\ \beta = \sigma^2 AB/I_2,\ \sigma z_k = v w_k$, (with $\sigma < \cdot >= v <'>$), we obtain the following analytic dynamic system (reference system) of the fourth order:

$$\begin{aligned}&\alpha' = -(1+H_1)w_2 + \beta\sin\alpha,\ w_2' = \beta\sin\alpha\cos\alpha - (1+H_1)w_1^2\frac{\cos\alpha}{\sin\alpha} - H_1 w_2\cos\alpha,\\ &w_1' = (1+H_1)w_1 w_2\frac{\cos\alpha}{\sin\alpha} - H_1 w_1\cos\alpha,\end{aligned} \tag{2}$$

$$\beta_1' = (1+H_1)w_1\frac{\cos\alpha}{\sin\alpha}, \tag{3}$$

which includes the independent third-order subsystem (2).

If $\beta = H_1$ then after the change of variables $w^* = \ln|w_1|$, the divergence of the right-hand side of (2) ((2), (3)) will become identically equal to zero, which allows considering the system(s) to be conservative.

2.2 Stability of Translational Motion

Consider the following positive definite function in the phase space of the third-order system (2):

$$V(\alpha, w_1, w_2) = w_2^2 + (1+\beta)w_1^2 + \beta[w_2 - \sin\alpha]^2. \tag{4}$$

Theorem. Function (4) is a Lyapunov (Chetaev) function for system (2), i.e., its derivative is negative definite for $\beta < H_1$ and positive definite for $\beta > H_1$.

Corollary. The origin of coordinates of system (2) (after the right-hand side is redefined at it) is an attracting singular point for $\beta < H_1$ and a repulsing singular point for $\beta > H_1$.

Indeed, by virtue of (2), the derivative of function (4) is $2(\beta - H_1)\cos\alpha[w_1^2 + w_2^2]$.

Note once again that a similar theorem is also valid for the general system with arbitrary influence functions y_N, z_N, and s.

References

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