PAMM · Proc. Appl. Math. Mech. **12**, 43 – 44 (2012) / **DOI** 10.1002/pamm.201210013

Cases of Complete Integrability in Transcendental Functions in Dynamics and Certain Invariant Indices

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The results of this work appeared in the process of studying a certain problem on the rigid body motion in a medium with resistance, where we needed to deal with first integrals having nonstandard properties. Precisely, they are not analytic, not smooth, and on certain sets, they can be even discontinuous. Moreover, they are expressed through a finite combination of elementary functions. However, the latter circumstances allowed us to carry out a complete analysis of all phase trajectories and show those their properties which have a "roughness" and are preserved for systems of a more general form having certain symmetries of latent type.

1 Introduction

The complete integrability of those systems is related to symmetries of a latent type. Therefore, it is of interest to study sufficiently wide classes of dynamical systems having analogous latent symmetries.

As it is known, the concept of integrability is sufficiently broad and undeterminate in general. In its construction, it is necessary to take into account in what sense it is understood (it is meant that a certain criterion according to which one makes a conclusion that the structure of trajectories of the dynamical system considered is especially attractive), in which function classes the first integrals are sought for, etc.

In this activity, the author applies such an approach such that as first integrals, transcendental functions are elementary. Here, the transcendence is understood not in the sense of elementary functions (e.g., trigonometrical functions) but in the sense that they have essentially singular points (by the classification accepted in the theory of functions of one complex variable according to which a function has essentially singular points). In this case, it is necessary to continue them formally to the complex plane. As a rule, such systems are strongly nonconservative.

2 Some Case of Transcendental Integrability

We will develop the qualitative methods in the theory of nonconservative systems that arise, e.g., in such fields of science as the dynamics of a rigid body interacting with a resisting medium, oscillation theory, etc. This material can call the interest of specialists in the qualitative theory of ordinary differential equations, in rigid body dynamics, as well as in fluid and gas dynamics since the work uses the properties of motion of a rigid body in a medium under the streamline flow around conditions [1, 2].

The author obtains new families of phase portraits of systems with variable dissipation on lower- and higher-dimensional manifolds. He discusses the problems of their absolute or relative roughness, He discovers new integrable cases of the rigid body motion, including those in the classical problem of motion of a spherical pendulum placed in the over-running medium flow [3].

To understand the difficulty of problem resolved, for instance, let us consider the spherical pendulum (ψ and θ — the coordinates of point on the sphere where the pendulum is defined) in a jet flow. Then the equations of its motion are

$$\ddot{\theta} + (b_* - H_1^*)\dot{\theta}\cos\theta + \sin\theta\cos\theta - \dot{\psi}^2\frac{\sin\theta}{\cos\theta} = 0, \tag{1}$$

$$\ddot{\psi} + (b_* - H_1^*)\dot{\psi}\cos\theta + \dot{\theta}\dot{\psi}\frac{1+\cos^2\theta}{\cos\theta\sin\theta} = 0,\ b_* > 0,\ H_1^* > 0, \tag{2}$$

and the phase pattern of the eqs. (1), (2) is on the Fig. 1.

The assertions obtained in the work for variable dissipation system are a continuation of the Poincare–Bendixon theory for systems on closed two-dimensional manifolds and the topological classification of such systems.

The problems considered in the work stimulate the development of qualitative tools of studying, and, therefore, in a natural way, there arises a qualitative variable dissipation system theory.

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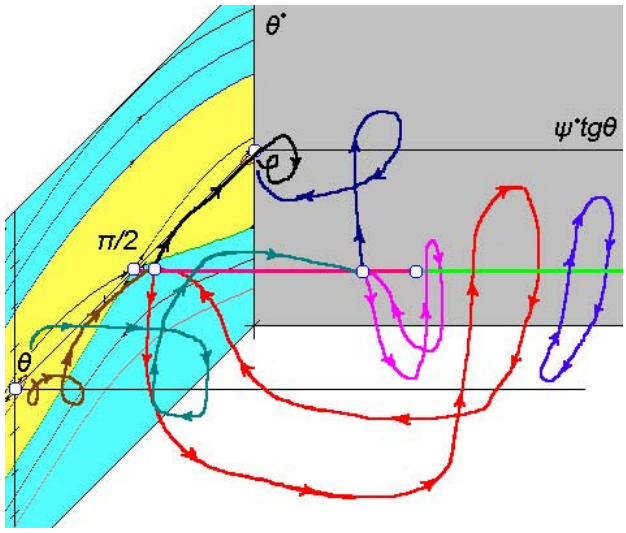

Fig. 1: Phase pattern of spherical pendulum in a jet flow.

Following Poincare, we improve some qualitative methods for finding key trajectories, i.e., the trajectories such that the global qualitative location of all other trajectories depends on the location and the topological type of these trajectories. Therefore, we can naturally pass to a complete qualitative study of the dynamical system considered in the whole phase space. We also obtain condition for existence of the bifurcation birth stable and unstable limit cycles for the systems describing the body motion in a resisting medium under the streamline flow around. We find methods for finding any closed trajectories in the phase spaces of such systems and also present criteria for the absence of any such trajectories. We extend the Poincare topographical plane system theory and the comparison system theory to the spatial case. We study some elements of the theory of monotone vector fields on orientable surfaces which form the so called *invariant indices of relatively structural stable vector fields* from dynamics of a rigid body interacting with a medium.

3 Complete Integrability of Certain Classes of Nonconservative Systems

Also, we show that for homogeneous circular cylinders moving in the water, the rectilinear translational drag is not stable for any dynamical and geometric parameters of such cylinders. Probably, this is related to the motion of the cylinders in the water, when the water damping is inessential, which does not allow us to speak about the stability of the rectilinear translational damping. However, for cylinders having a hole in their interior, the attainment of the above stability is possible under certain conditions.

Therefore, under certain conditions, the account for the medium damping action on a rigid body leads to an affirmative answer to the principal question of nonlinear analysis: under the body motion in a medium with finite angles of attack, in principle, the appearance of stable auto-oscillations is possible. Moreover, for circular cylinder, the appearance of stable and unstable auto-oscillations is possible!

All what said above, allows us to estimate the results of the work as a whole as a new direction in analytical mechanics of a rigid body interacting with a medium.

As is known, the concept of integrability is sufficiently broad and indefinite in general. In its constructing, it is necessary to take into account what it means (we mean a certain criterion according to which one makes a conclusion about that the structure of trajectories of the system considered is especially "attractive and simple"), in which function classes the first integrals are sought for.

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