Variety of the Cases of Integrability in Dynamics of a 2D- and 3D-Rigid Body Interacting with a Resisting Medium

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ABSTRACT

This work is devoted to the development of qualitative methods in the theory of nonconservative systems that arise, e.g., in such fields of science as the dynamics of a rigid body interacting with a resisting medium, oscillation theory, etc. This material can call the interest of specialists in the qualitative theory of ordinary differential equations, in rigid body dynamics, as well as in fluid and gas dynamics since the work uses the properties of motion of a rigid body in a medium under the streamline flow around conditions [1].

The author obtains a full spectrum of complete integrability cases for nonconservative dynamical systems having nontrivial symmetries. Moreover, in almost all cases of integrability, each of the first integrals is expressed through a finite combination of elementary functions and is a transcendental function of its variables, simultaneously. In this case, the transcendence is meant in the complex analysis sense, i.e., after the continuation of the functions considered to the complex domain, they have essentially singular points. The latter fact is stipulated by the existence of attracting and repelling limit sets in the system considered (for example, attracting and repelling foci) [2].

The author obtains new families of phase portraits of systems with variable dissipation on lower- and higher-dimensional manifolds. He discusses the problems of their absolute or relative roughness, He discovers new integrable cases of the rigid body motion, including those in the classical problem of motion of a spherical pendulum placed in the over-running medium flow [3].

To understand the difficulty of problem resolved, for instance, let us consider the spherical pendulum (ψ and θ — the coordinates of point on the sphere where the pendulum is defined) in a jet flow. Then the equations of its motion are

$$\ddot{\theta} + (b_* - H_1^*)\dot{\theta}\cos\theta + \sin\theta\cos\theta - \dot{\psi}^2\frac{\sin\theta}{\cos\theta} = 0, \quad (1)$$

$$\ddot{\psi} + (b_* - H_1^*)\dot{\psi}\cos\theta + \dot{\theta}\dot{\psi}\frac{1+\cos^2\theta}{\cos\theta\sin\theta} = 0,\ b_* > 0,\ H_1^* > 0, \quad (2)$$

and the phase pattern of the eqs. (1), (2) is on the Fig. 1.

The assertions obtained in the work for variable dissipation system are a continuation of the Poincare–Bendixon theory for systems on closed two-dimensional manifolds and the topological classification of such systems.

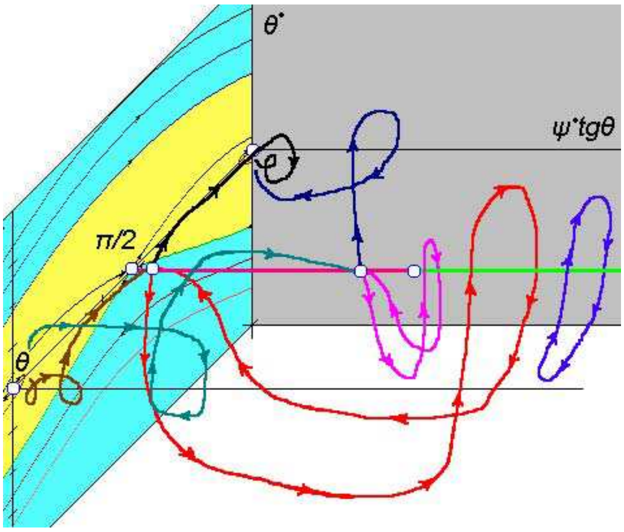

Figure 1: Phase pattern of spherical pendulum in a jet flow.

The problems considered in the work stimulate the development of qualitative tools of studying, and, therefore, in a natural way, there arises a qualitative variable dissipation system theory.

Following Poincare, we improve some qualitative methods for finding key trajectories, i.e., the trajectories such that the global qualitative location of all other trajectories depends on the location and the topological type of these trajectories. Therefore, we can naturally pass to a complete qualitative study of the dynamical system considered in the whole phase space. We also obtain condition for existence of the bifurcation birth stable and unstable limit cycles for the systems describing the body motion in a resisting medium under the streamline flow around. We find methods for finding any closed trajectories in the phase spaces of such systems and also present criteria for the absence of any such trajectories. We extend the Poincare topographical plane system theory and the comparison system theory to the spatial case. We study some elements of the theory of monotone vector fields on orientable surfaces.

References

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