CASES OF INTEGRABILITY IN DYNAMICS OF A RIGID BODY INTERACTING WITH A RESISTANT MEDIUM

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Summary The author discovers new integrable cases of the rigid body motion in a resisting medium, including those in the classical problem of motion of a spherical pendulum placed in the over-running medium flow. In this work, we study the problem on the body motion under the condition that the line of the force applied to the body does not change its orientation with respect to the body and can only displace parallel to itself depending on the angle of attack and, possibly, on other phase variables.

SEQUENCE OF STEPS IN MODELLING

The process of describing the force field is a sequence of steps. We first study a preparatory model of the force field and construct a family of mechanical systems whose motion has different characteristics that essentially depend on model parameters such that the information about them is incomplete or does not exist at all. As a result of studying such a model, there arise questions such that the answers to them cannot be found in the framework of the accepted model. Then the elaborated objects become the subject of a detailed experimental study at the second step. Such an experiment presupposes the answers to the formulated questions and either introduces necessary corrections to the preparatory constructed model or reveals new questions, which lead to the necessity of the first step repetition but in a new level of the problem understanding. Sometime, we can succeed in obtaining the answers to questions of qualitative character when discussing the traditional problem of analytic mechanics, the problem of existence of the full tuple of first integrals for the constructed dynamical system. At the same time, the study of the behavior of a dynamical system "as a whole" often forces us to use the numerical experiment. In this case, there arises the necessity of elaborating new computational algorithms or improving the known, as well as new qualitative methods.

In this work, we study the problem on the body motion under the condition that the line of the force applied to the body does not change its orientation with respect to the body and can only displace parallel to itself depending on the angle of attack and, possibly, on other phase variables. Such conditions arise under the plate motion with the so-called "large" angles of attack in a medium under a streamline flow (in this case, the fluid is assumed to be ideal in general, although all this are also true for fluids of a small viscosity, first of all, for the water) or under a separation flow (which is justified by an experiment completely satisfactory).

Physical assumptions and quasi-stationarity hypothesis

Assume that a rigid body of mass m executes a plane-parallel (spatial) motion in a medium with quadratic resistance law and that a certain part of the exterior body surface is a plane plate being under the medium streamline flow conditions. This means that the action of the medium on the plate reduces to the force $\mathbf{S}$ (applied at the point N) whose line of action is orthogonal to the plate. Let the remained par of the body surface be situated in a volume bounded by the flow surface that goes away from the plate boundary and is not subjected by the medium action. For example, similar conditions can arise after the body entrance into the water. Assume that among the body motions, there exists a rectilinear plane-parallel drag regime. This in possible when the following two conditions hold: 1) the body velocity is orthogonal to the plate; 2) the perpendicular dropped from the body center of gravity C on the plate plane belongs to the line of the action of the force $\mathbf{S}$.

Let us relate to the body the right coordinate system $Dxyz$ whose axis z moves parallel to itself, and for simplicity, assume that the plane Dzx is the geometric symmetry plane of the body. This ensures the fulfillment of property 2) under the motion satisfying condition 1). To construct the dynamical model, let us introduce the following phase coordinates: the value $v = |\mathbf{v}|$ of the velocity $\mathbf{v}$ of the point D, the angle α between the vector $\mathbf{v}$ and the axis x, and the algebraic value Ω of the projection of the body absolute angular velocity on the axis z.

Assume that the value of the force $\mathbf{S}$ quadratically depend on v with nonnegative coefficient s_1 ($S = s_1 v^2$). As usual, one represents s_1 in the form $s_1 = \rho P c_x/2$, where c_x is now the dimension-free coefficient of the frontal resistance (ρ is the medium density and P is the plate area). This coefficient depends on the angle of attack, the *Struchal number*, and other quantities which are usually considered as parameters. In what follows, we also introduce the following additional phase variable of the "Struchal type"; $\omega = \Omega D/v$, where D is the characteristic plate transversal size. We restrict ourselves to the dependence of c_x on the pair (α, ω) of variables, i.e., we assume that s_1 (as well as y_N) is a function of the pair (α, ω) of dimension-free variables.

Let us define (purely formally for now) the dependence of s_1 and the ordinate y_N of the point N on the phase coordinates (α, ω). The system of dynamical equations must admit a particular solution of the form $\alpha(t) \equiv 0,\ \omega(t) \equiv 0$. Therefore, we have the condition $y_N(0,0) = 0$ for the function $y_N(\alpha, \omega)$, and in the linear case, we need to assume that $y_N = D(k\alpha - h\omega)$, where k and h are certain constants. Because the approximation is linear, we can ignore the dependence of s_1 on α and ω.

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IMPORTANT SAMPLE

As a sample of this approach we give the following notes. The author obtains a full spectrum of complete integrability cases for nonconservative dynamical systems having nontrivial symmetries. Moreover, in almost all cases of integrability, each of the first integrals is expressed through a finite combination of elementary functions and is a transcendental function of its variables, simultaneously. In this case, the transcendence is meant in the complex analysis sense, i.e., after the continuation of the functions considered to the complex domain, they have essentially singular points. The latter fact is stipulated by the existence of attracting and repelling limit sets in the system considered (for example, attracting and repelling foci) [1], [2].

The author obtains new families of phase portraits of systems with variable dissipation on lower- and higher-dimensional manifolds. He discusses the problems of their absolute or relative roughness, He discovers new integrable cases of the rigid body motion, including those in the classical problem of motion of a spherical pendulum placed in the over-running medium flow [3].

To understand the difficulty of problem resolved, for instance, let us consider the spherical pendulum (ψ and θ — the coordinates of point on the sphere where the pendulum is defined) in a jet flow. Then the equations of its motion are

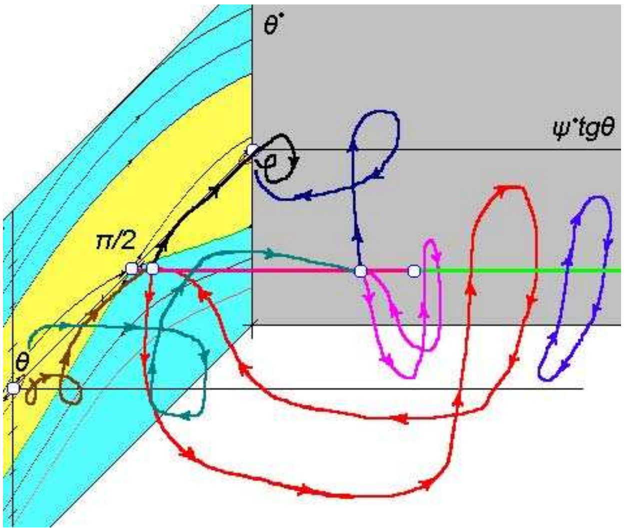

Figure 1. Phase pattern of spherical pendulum in a jet flow.

$$\ddot{\theta} + (b_* - H_1^*)\dot{\theta}\cos\theta + \sin\theta\cos\theta - \dot{\psi}^2\frac{\sin\theta}{\cos\theta} = 0, \quad (1)$$

$$\ddot{\psi} + (b_* - H_1^*)\dot{\psi}\cos\theta + \dot{\theta}\dot{\psi}\frac{1+\cos^2\theta}{\cos\theta\sin\theta} = 0,\ b_* > 0,\ H_1^* > 0, \quad (2)$$

and the phase pattern of the eqs. (1), (2) is on the Fig. 1.

CONCLUSIONS

Following Poincare, we improve some qualitative methods for finding key trajectories, i.e., the trajectories such that the global qualitative location of all other trajectories depends on the location and the topological type of these trajectories. Therefore, we can naturally pass to a complete qualitative study of the dynamical system considered in the whole phase space. We also obtain condition for existence of the bifurcation birth stable and unstable limit cycles for the systems describing the body motion in a resisting medium under the streamline flow around. We find methods for finding any closed trajectories in the phase spaces of such systems and also present criteria for the absence of any such trajectories. We extend the Poincare topographical plane system theory and the comparison system theory to the spatial case. We study some elements of the theory of monotone vector fields on orientable surfaces.

References

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