STRUCTURAL OPTIMIZATION OF THE CONTROLLED RIGID MOTION IN A RESISTING MEDIUM

MAXIM V. SHAMOLIN

Institute of Mechanics, Lomonosov Moscow State University, 1 Michurinsky Prospect, 119899 Moscow, Russia

ABSTRACT

This paper is devoted to three-dimensional structural optimization of some classes of controlled motions of a rigid body in a resisting medium. It is more important to consider the most interesting case of motion - the deceleration of a rigid body in a medium. Given investigation is located in the connection of a qualitative theory of ordinary differential equations, rigid body dynamics and can be considered with the dynamics of a jet flow. The main purpose of this paper is to spread the results from plane-parallel dynamics to three-dimensional case of motions.

KEYWORDS

Structural Optimization, Resisting Medium, Deceleration

INTRODUCTION

A mathematical model is constructed describing the deceleration of a solid body moving in a medium with a jet flow around the body in three-dimensional space. The regime of translational deceleration is shown to be normally unstable. This has made it possible to develop a relatively simple technique for determining model parameters experimentally. An example of the application of this technique to a cylindrical body is presented.

The deceleration problem turned out to be more convenient for checking the build-up effect by experiment. The present study made it possible to develop a fairly simple and efficient technique for determining the unknown parameters of the model.

Statement of a problem about the motion of a rigid body in a resisting medium when all the conditions of jet or separated flow are satisfied is given. This interaction of a medium with a body is concentrated on that part surfaces of a body, which has the form of a flat area.

At formation of dynamic model of influence of a medium on a body some properties of a medium are marked and its have connected in a serie of hypothesises. And the basic hypothesis is a hypothesis of quasistationarity. In this connection complete dynamic system describing investigated model is shown. Such class of motions which allow some constrain is considered. That constrain is allowing to consider some quantity as a constant in all time of a motion. That quantity is size of velocity of some characteristic point of a rigid body. The qualitative analysis of dynamic system obtained in space of quasivelocities is presented and it is recognized all the non-linear non-trivial properties.

BASIC HYPOTHESISES

It is offered some rather simple dynamic model of influence of a medium on a body. This model is not looked like traditional models the same as, used in aerodynamics. At formation of the given dynamic model the basic stages of so-called phenomenological view-point were found their reflection, namely, some experimental facts, the well-known solution of exact-stated problems and etc., and also additional hypothesises were considered.

At formation of dynamic model of influence of a medium on a body were taken into account both a further undersanding of experimental dates and prediction of the new facts, delimitation of applicability of experiment and other [1-4]. Such modelling is connected to a problem, which is arising behind the researcher in connection with a motion of a rigid body and a medium. Therefore for the same body (or for the same class of motions) the model of influence on it can be different from a medium.

We shall note two basic properties of a medium. The first property is "the inertia of a medium". External forces (including the resistant force) applied to a body, inform acceleration not only the body, but also the particles of a medium. The second property is "the resistance of a medium". If the external forces do not act on a body moving in a silent medium and there is no any internal sources of energy inside the body then the velocity of a body will decrease in time and early or latly the body will stop. At the same time a medium will be "silent" with the body. The exception is made only by an ideal liquid (the well-known paradox of D'Alambert-Euler), but at construction of model of a jet flow usually the viscous medium is considered.

Alongside with hypothesises of convertibility (this is connected to a principle of relativity of Galiley) and "the connection" one on other two listed above properties of medium, at construction of this model of influence of a medium on a body the hypothesis of quasistationarity has the basic role. As it is known, this hypothesis is widely used in applied aerodynamics and this is connected with two mentioned above properties of medium. The basis of a hypothesis of quasistationarity is such that the distribution of the velocities of particles of a medium is accepted such, which it would have a place at stationary movement of a body. Thus, a medium "remembers" only the instant movement of a body and "forgets" its entry conditions [1-4].

FORMULATION OF THE PROBLEM

Suppose that the solid body with the mass *m* executes a motion in a medium offering square-law resistance. We shall assume that part of the ourter surface of the body is a flat area Π in a jet flow of the medium. This means that the action of the medium on the area can be reduced to the force **S**, whose line of action is orthogonal to the plate of area Π. We shall also assume that the rest of the body surface is contained within a volume bounded by the surface of the jet shedding from the plate edge, so that it does not experience the resistance of the medium. Such conditions can occur, for example, after a body penetrates into a fluid [2,4].

Let the regime of rectilinear translational deceleration exist among the possible body motions. This is possible if: (1) the velocity of the body motion is normal to the plate Π, and (2) the perpendicular *DC* dropped from the center of gravity *C* of the body onto the plane Π is parallel to the line of action of the force **S**.

Let a right-handed system of coordinates *Dxyz* be fitted to the body, the *y* axis moving parallel to itself; for simplicity, we shall assume that the *Dy* and *Dz* axes (its belong the plane Π) likely *Dx* are the main axises of inertia, and *Dx* = *DC*. Obviously, this ensures the fulfillment of conditions (2) in a motion satisfying condition (1).

Let us study the problem of the stability of the body's rectilinear translational deceleration. To construct a dynamic model of the motion, we introduce the phase coordinates: *v*, the magnitude of the velocity of the point *D*, *a*, the angle between the vector **v** and the *x* axis, *b*, the angle between the projection of vector **v** onto the plane *Dyz* and *y* axis, and *p*,*q*,*r*, projections of the absolute velocity of the body onto system of coordinates *Dxyz*.

We write down the equations governing the body motion as the equations of the motion of its center of mass projected onto the *x*,*y*,*z* axes, together with the equations governing the variation of the kinetic moment with respect to the *x*,*y*,*z* axes:

$$v'cosa - a'vsina + qvsinasinb - rvsinacosb + l(q^2 + r^2) = - S/m$$

$$v'sinacosb + a'vcosacosb - b'vsinasinb + rvcosa - pvsinasinb - lpq - lr' = 0 \qquad (1)$$

$$v'sinasinb + a'vcosasinb + b'vsinacosb + pvsinacosb - qvcosa - lpr + lq' = 0$$

$$Ap' + (C - B)qr = 0$$

$$Bq' + (A - C)pr = - RSsinb \qquad (2)$$

$$Cr' + (B - A)pq = RScosb$$

Here, *l* = *DC*, *A*,*B*,*C* are the main moments of inertia of the body, and *R* is the distanse from *D* point to the point of intersection of the plate Π and the line of action of the force **S**. We shall consider the case of dynamical symmetry

$$B = C.$$

Thus, there is exist the cyclic integral

$$p = p_0$$

in the system. For simplicity, study the case when the constant value p_0 is equal to zero.

To complete the description of the model, we shall use the quasi-stveady-state hypothesis [1], that is, we shall assume that $S = sv^2$ and preassign the dependence of the quantities s and R on the phase coordinates a and p,q,r. The system of equations *(1), (2)* must admit the particular solution $a(t) = p(t) = q(t) = r(t) = 0$. Hence we have $R(0,0,0,0) = 0$ for the function $R(a,p,q,r)$.

LINEARIZED MODEL AND THE PROBLEM OF STABILITY

We shall examine the problem of the stability of the trivial solution on the basis of the first approximation in the variables a and p,q,r. Then for R we have: $R = d(ka - h_1dz_1/v - h_2dz_2/v)$, where d is the diameter of the area Π, $z_1 = qcosb + rsinb$, $z_2 = rcosb - qsinb$.

The linearized model of the force exerted by the medium contains four parameters s, k, h_1, and h_2, which are determined by the shape of the plate in plan. The parameters k, h_1, and h_2, are dimensionless; they can be determined by balance measurements in experimental installations such as hydrodynamic and aerodynamic tunnels. For certain types of plates, these quantities have also been determined theoretically [2]; in that case, $k > 0$. The necessity of introducing the parameters h_1 and h_2 into the model is not obvious a priori.

The parameters k, h_1 and h_2 are related to the so-called "rotary derivatives of the moment"; only scanty information on these derivatives exists in the literature. This is given impetus to an investigation, one of whose preliminary results was reported in [4].

Correct to the terms linear in a and p,q,r, equations *(1)* and *(2)* take the form

$$v' = - sv^2/m$$

$$a' + z_2 - sva/m - ldsv/B[ka - h_1z_1d/v - h_2z_2d/v] = 0$$

$$z_1' = b'z_2$$

$$z_2' = - b'z_1 + v^2ds/B[ka - h_1z_1d/v - h_2z_2d/v]$$

Introducing, as is usual for systems of this kind, the natural parameter i, $v(diff)t = d(diff)i$, changing the variable $Z_k = dz_k/v$ and using an obvious differentiation formula, we obtain the following system:

$$v' = - sv/m$$

$$a' = - Z_2[1 + sld^2h_2/B] - Z_1sld^2h_1/B + sd[1/m + kld/B]a$$

$$Z_1' = Z_1 sd/m,\ Z_2' = Z_2[1- md^2h_2/B]ds/m - Z_1d^3sh_1/B + sd^3ka/B$$

where the derivatives with respect to i are dotted. The group of the last equations do not contain the quantity v and can be examined independently of another equations of the system (it is correctly when, of cource, $p = 0$ in all moments of time; in general, the quantity p stays as constant in all moments of time because our body is dynamically symmetric and the force **S** is perpendicular to the plate).

The resulting unstable motion can be used to determine the unknown parameters and, in particular, k and h_1, and h_2.

The nonlinear case of this investigation was considered in [5-7]. The plane-parallel case was studied in [8]. The full non-linear qualitative theory of such systems are developed in [9-16] and in many other works.

The work was financially supported by the Russian Foundation for Fundamental Research in 1993-1996.

REFERENCES

[1] Chaplygin, S.A. *Selected Works*, Nauka, Moscow, 1976.

[2] Eroshin, V.A., Privalov, V.A., & Samsonov, V.A. Two model problems of the motion of a body in a resisting medium, *Collection of Scientific and Methodological Papers on Theoretical Mechanics*, 1987, **18** [in Russian], 75-78.

[3] Eroshin, V.A., Samsonov, V.A., & Shamolin, M.V. A model problem of deceleration of a body in a resisting medium with a jet flow around the body, *Fluid Dynamics*, 1995, Vol.30, **3**, 351-355.

[4] Gurevich, M.I. *Theory of Ideal Fluid Jets*, Nauka, Moscow, 1979.

[5] Lokshin, B.Ya., Privalov, V.A., & Samsonov V.A. *Introduction to the Problem of the Motion of a Body in a Resisting Medium* [in Russian], Moscow Univ. Press, Moscow, 1986.

[6] Shamolin, M.V. Relative structural stability on the problem of a body motion in a resisting medium, p.207, *Abstracts of Short Communications of Int. Cong. of Math.*, Zurich, Switzerland, 1994, Zurich, Switzerland, 1994.

[7] Shamolin, M.V. New two-parameter families of the phase patterns on the problem of a body motion in a resisting medium, p.436, *Book of Abstracts of Int. Congr. in Indust. & Appl. Math.*, Hamburg, Germany, 1995, Hamburg, Germany, 1995.

[8] Shamolin, M.V. Qualitative methods to the dynamic model of an interaction of a rigid body with a resisting medium and new two-parametric families of the phase portrairs, p.185, *Abstracts of Dyn. Days Workshop*, Lyon, France, 1995, Lyon, France, 1995.

[9] Shamolin, M.V. Introduction to the problem of a rigid body deceleration in a resisting medium and new two-parametric families of phase portraits, *Moscow University Mechanics Bulletin*, 1996, **4** [in Russian], p.57-69.

[10] Shamolin, M.V. On some case of integrability in three-dimensional rigid body dynamics when a rigid body is interacted with a resisting medium, p.91-92 [in Russian], *Abstracts of II Simposium in Clas. & Celest. Mech.*, Velikiye Luki, 1996, Moscow-Velikye Luki, 1996.

[11] Shamolin, M.V. Qualitative methods in dynamics of a rigid body interacting with a medium, part.III, p.267 [in Russian], *Abstracts of II Sibirian Congr. in appl. & industr. Math.,* Novosibirsk, 1996, Novosibirsk, 1996.
[12] Shamolin, M.V. Manifold of types of phase portraits in dynamics of a rigid body interacting with a medium, *Physics Doklady,* 1996, **349** [in Russian], No.2, p.193-197.
[13] Shamolin, M.V. Qualitative methods in interacting with the medium rigid body dynamics, p.129-130, *Abstracts of GAMM Wissenschaftliche Jahrestangung'96,* Prague, Czech Rep., 1996, Prague, Czech Rep., 1996.
[14] Shamolin, M.V. Definition of Relative Structural Stability and two-parametric family of phase portraits in rigid body dynamics, *Mathematical Surveys*, 1996, **51** [in Russian], No.1, p.175-176.
[15] Shamolin, M.V. On relative structural stability of dinamical systems in the problem of a body motion in a resisting medium, *Moscow University Mathematics Bulletin,* 1995, **6** [in Russian], p.17.
[16] Shamolin, M.V. Poisson-stable and dense orbits in rigid body dynamics, Abstracts of 3rd Experimental Chaos Conference, Edinburg, Scotland, 1995, Edinburg, Scotland, 1995.