Some problems of qualitative analysis in the modeling of the motion of rigid bodies in resistive media. (English summary)


The motion of a rigid body in a resistive medium is considered under a number of simplifying assumptions about the force interaction between the body and the medium. For instance, it is assumed that the moving flow pushes the body’s flat frontal part but has no force interaction with the rest of the body’s surface. Sufficient conditions for the asymptotic stability of the rectilinear translational deceleration are obtained. It is found that under certain conditions associated with higher derivatives of the influence function (the arm of the influence force and the resistance coefficient), stable or unstable self-oscillating modes of the body motion can arise. For a two-dimensional cylinder a new multiparameter family of phase portraits of self-oscillating modes in the restricted region of the angle of attack is specified. For homogeneous circular cylinders some estimates of their inertia and mass are given based on some experiments.

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12. V. A. Eroshin, V. A. Privalov, and V. A. Samsonov, “Two model problems of body


52. M. V. Shamolin, “On the problem of the motion of the body with front flat butt end in a resistive medium,” in: *Scientific Report of Institute of Mechanics, Moscow State University* [in Russian], No. 5052, Institute of Mechanics, Moscow State University, Moscow (2010).


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Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)
New cases of integrability of equations of motion of a rigid body in the n-dimensional space.

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tion in Rigid Body Dynamics [in Russian], Ekzamen, Moscow (2007).


67. M. V. Shamolin, “Variety of the cases of integrability in dynamics of a 2D- and 3D-
rigid body interacting with a medium,” in: 8th ESMC 2012, CD-Materials (Graz, Austria, July 9–13, 2012), Graz, Graz, Austria (2012). MR2963695

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

Andreev, A. V. (RS-RPF-NDM);
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)
Methods of mathematical modeling of the action of a medium on a conical body.
(English summary)

The authors consider a mathematical model of a plane-parallel action of a medium on a rigid body whose surface has a section, which is a circular cone. They derive a complete system of equations of motion under the quasi-stationarity conditions in a plane-parallel motion of a homogeneous rigid body of mass $m$ with the cone-shaped front part interacting with a flow of medium under conditions of jet circumfluence. The forces of frontal and side resistance are quadratic functions of the body speed. The first two equations are obtained from the theorem on the motion of the mass center and the third from the theorem on the change of the angular momentums in the König axes. The position coordinates in the system are cyclic. The system of kinematic equations completes the mathematical model. To obtain the form of the functions $R(\alpha)$, $s(\alpha)$, and $b(\alpha)$, which appear in the equations, one needs experimental information on the properties of jet circumfluence. The authors discuss possible classes of these functions. Among all possible motions of a body, there exists a key regime a rectilinear translational deceleration: the body moves translationally with zero attack angle $\alpha$ and speeds of all points of the body decrease. The key regime corresponds to the trivial solution of the system. From the point of view of the stability theory, this type of stability is treated as stability with respect to only some of the variables. The authors consider the case where the system contains two force couples: a couple of frontal resistance forces and a couple of lateral forces. They show that by an appropriate choice of the corresponding initial conditions one can obtain a conditionally stable solution. As a result they obtain an infinite family of phase portraits on the phase cylinder of quasi-velocities corresponding to the presence in the system only of a nonconservative pair of forces. The family of portraits deals with the case of the conditional stability, which can be achieved by an appropriate choice of initial conditions.

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No. 1, 919–975 (2003). MR1965083


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16. M. V. Shamolin, “Classification of complete integrability cases in four-dimensional symmetric rigid-body dynamics in a nonconservative field” [in Russian], Sovrem.
Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)

On the problem of free deceleration of a rigid body with a leading cone in a resisting medium. (Russian. English, Russian summaries)
Mat. Model. 28 (2016), no. 9, 3–23.

Summary: “The author constructs the nonlinear mathematical model of the planar interaction of a medium to the rigid body having the circular convex as the front part of its external shape. We make the multi-parametric analysis of dynamic equations of the body motion. We obtain new family of the phase patterns on the phase cylinder of quasi-velocities. This family consists of the infinite set of topologically nonequivalent phase patterns. We also obtain the sufficient conditions of important regime stability, i.e. the rectilinear translational deceleration, and also the conditions of existence of auto-oscillations in the system considered.”

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-NDM)

Integrable nonconservative dynamical systems on the tangent bundle of the multidimensional sphere. (English summary)

In this paper, the author constructs a class of nonconservative systems of differential equations on the tangent bundle of the sphere of any finite dimension. This class has a complete set of first integrals, which can be expressed as finite combinations of elementary functions. Most of these first integrals consist of transcendental functions of their phase variables. In this article a transcendental function is a function with essentially singular points, as in the theory of functions of a complex variable.

Camelia M. Frigioiu

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.
Cases of integrability in the dynamics of a multidimensional rigid body in a nonconservative field in the presence of a tracking force. (Russian. English, Russian summaries)


Summary: “This paper is a survey of integrable cases in the dynamics of a five-dimensional rigid body under the action of a nonconservative force field. We review both new results and results obtained earlier. Problems examined are described by dynamical systems with so-called variable dissipation with zero mean. The problem of the search for complete sets of transcendental first integrals of systems with dissipation is quite topical; a large number of works are devoted to it. We introduce a new class of dynamical systems that have a periodic coordinate. Due to the existence of nontrivial symmetry groups of such systems, we can prove that these systems possess variable dissipation with zero mean, which means that on the average for a period with respect to the periodic coordinate, the dissipation in the system is equal to zero, although in various domains of the phase space, either the energy pumping or dissipation can occur. Based on the facts obtained, we analyze dynamical systems that appear in the dynamics of a five-dimensional rigid body and obtain a series of new cases of complete integrability of the equations of motion in transcendental functions that can be expressed through a finite combination of elementary functions.”

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68. M. V. Shamolin, “New cases of the complete integrability in the dynamics of a


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-NDM)
Integrable systems with variable dissipation on the tangent bundle of a multidimensional sphere and applications. (Russian. English, Russian summaries)

In this paper the author presents a survey of integrable cases in dynamics of a two-, three- or four-dimensional rigid body that moves in a nonconservative force field under the so-called “variable dissipation with zero mean”. Almost all the results belong to the author.

In part 1 the general concepts for dynamics of a free \( n \)-dimensional rigid body are introduced (tensor of angular velocity of the body, dynamical equations of motion of the body in \( \mathbb{R}^n \times \text{SO}(n) \), the generalized Euler and Rivals formulae and tensor of inertia of the body). The body moves in a resistant medium that produces a following nonconservative resultant force under action of which the body’s characteristic point has a constant velocity or the body has a uniform motion of its center of mass. In parts 1 and 2 the complete sets of constants of motion in these two cases are given.

In part 3 the case of a 4-dimensional rigid body is systematically studied. The results pertain to the case in which the interaction between the body and the medium concentrates at that part of the body surface which has the form of a 2-dimensional disk.

In part 4 the qualitative analysis of the plane and spatial problems for motion of real rigid bodies in a resistant medium is proposed. The nonlinear model of action of the medium upon the rigid body is constructed using the dependence of the force moment on the reduced angular velocity of the body. Some problems of stability for the basic types of the body motions are also considered. Part 5 gives data for conducting natural experiments on the motion of free hollow circular cylinders in a resistant medium.

A. S. Sumbatov
A complete list of the first integrals of the motion equations of a multidimensional rigid body in a nonconservative field in the presence of linear damping. (Russian. Russian summary)


The motion of a certain $n$-dimensional dynamically symmetrical rigid body (the body has only two different moments of inertia) under action of potential forces, with a tracking force that provides the fulfillment of one kinematic condition and a variable linear damping force, is considered. The principal point is postulating Newton’s second law for the case of a Euclidean space of any dimension. The complete set of the first integrals (cyclic or expressed in terms of elementary functions of auxiliary variables) is given.

Douglas S. Shafer

A complete list of the first integrals of the equations of motion of a multidimensional rigid body in a nonconservative field. (Russian)


In the $n$-dimensional space, the motion of a homogeneous rigid body in a non-conservative field is considered. In addition, a certain dynamical symmetry of the body is assumed together with the condition that the interaction of the medium with the body is concentrated in an $(n − 1)$-dimensional disk, which is a part of the body boundary.

Douglas S. Shafer
The author derives the full list of first integrals of that system. 

Andreev, A. V. (RS-RPF-NDM); Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)
Simulation of the action of a medium on a conical body and the family of phase portraits in the space of quasivelocities. (English summary)

The authors study the plane-parallel motion of a homogeneous rigid body with a conical front part interacting with the medium. Under quite general assumptions the equations of motion are reduced to a system of two nonlinear equations. Then the stability of an equilibrium point of the reduced system is investigated. The phase portraits presented show various dynamical behaviours of the system.

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-IMC)
A multidimensional pendulum in a nonconservative force field. (Russian)

The author studies the motion of an \( n \)-dimensional spherical pendulum in the \( n \)-dimensional Euclidean space under a constant flow that gives rise to some non-conservative field. After the introduction of generalized spherical coordinates \((\xi, \eta_1, \ldots, \eta_{n-2})\), the dynamics of the spherical pendulum can be reduced on the tangent bundle \( TS^{n-1} = \{(\xi^*, \eta^*_1, \ldots, \eta^*_{n-2}, \xi, \eta_1, \ldots, \eta_{n-2})\} \) of the \((n-1)\)-dimensional sphere:

\[
\begin{align*}
\dddot{\xi} + b \dot{\xi} \cos \xi + \sin \xi \cos \xi - \sum_{s=1}^{n-2} [\eta^*_s \sin \eta_1 \sin \eta_2 \cdots \sin \eta_{s-1}]^2 \tan \xi &= 0, \\
\dddot{\eta}_k + b \dot{\eta}_k \cos \xi + \dot{\xi} \eta^*_k \frac{1 + \cos^2 \xi}{\sin \xi \cos \xi} + 2 \eta^*_s \sum_{s=1}^{k-1} \eta^*_s \cot \eta_s - [\eta^*_k+1]^2 \\
&\quad - \sum_{s=2}^{n-k-2} [\eta^*_k+s \sin \eta_{k+1} \sin \eta_{k+2} \cdots \sin \eta_{k+s-1}]^2 \sin \eta_k \cos \eta_k = 0.
\end{align*}
\]

Here \( \dot{\cdot} \) denotes differentiation, \( k = 1, 2, \ldots, n-2 \), and the constant \( b \) depends on the non-conservative flow.
Finally, \( n \) independent first integrals of the above system of \((n-1)\) second-order ODE’s are found.

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-NDM)
Some classes of integrable problems in spatial dynamics of a rigid body in a nonconservative force field. (English summary)

This paper is a review of some previous and new results on integrable cases of a three-dimensional rigid-body motion in a nonconservative field.

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17. V. V. Golubev, *Lectures on the Integration of the Equations of Motion of a Heavy Solid Body with a Fixed Point* [in Russian], Gostekhizdat, Moscow (1953). MR0061942
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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)

New case of complete integrability of dynamics equations on a tangent fibering to a 3D sphere. (English summary)


The author presents the results of a study of the motion equations for a dynamically symmetric 4D-rigid body placed in a certain non-conservative field of forces. In this paper, a new case of integrability is obtained for dynamic equations of body motion...
in a resisting medium filling a four-dimensional space in the presence of a tracking force.  

Pavel Evgen’evich Ryabov

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<td>Simulation of rigid body motion in a resisting medium and analogies with vortex streets. (English summary)</td>
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The author brings together results from earlier papers on the motion of a 2-dimensional symmetric rigid body in a resisting medium modelled by a non-conservative force field. Additional solutions to the problem are found when the field also has dependence on the angular velocity of the body. Analogies of the resulting phase flows are drawn with Karman vortex streets.  

Peter S. Donelan

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<td>Classification of integrable cases in the dynamics of a four-dimensional rigid body in a nonconservative field in the presence of a tracking force. (English summary)</td>
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The paper investigates integrability conditions of special classes of non-conservative systems. In particular, for dynamic systems with a periodic coordinate, under a condition that dissipation vanishes in average over a period, the existence of first integrals is proven. This general result is used to classify integrable cases of a 4-dimensional rigid body in a non-conservative force field.  

Igor N. Nikitin

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of Motion of a Point and a Body in a Resisting Medium [in Russian], Moscow State Univ., Moscow (1992).


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Aidagulov, R. R. [A˘ ıdagulov, R. R.] (RS-MOSC-NDM); Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)

Topology on polynumbers and fractals.


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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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Aidagulov, R. R. [A˘ ıdagulov, R. R.] (RS-MOSC-NDM); Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)

Polynomials, norms, metrics, and polyingles.


The authors give some approaches to the notion of polyingles between $k$ vectors in a space with a metric of rank $k$, which generalizes the notion of angle (bingle) between two vectors for quadratic metrics.

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Shamolin, Maxim V. [Shamolin, Maxim V.] (RS-MOSC-IMC)

A new case of integrability in the dynamics of a multidimensional rigid body in a nonconservative field taking linear damping into account. (Russian)


The author studies the motions of a homogeneous n-dimensional rigid body with dynamical symmetry, under the action of a special non-conservative exterior force. It is supposed that there is a non-integrable constraint \( v = \text{const} \), where \( v \) is the norm of the velocity of a suitable point (center of mass of an \((n-1)\)-dimensional disc in the body, on which the non-conservative force acts). The first integrals, related to the dynamical symmetry (angular velocities), are also supposed to be zero.

Under these conditions a reduction of the initial system leads to a differential system on the tangent bundle of the \((n-1)\)-dimensional sphere \( S^{n-1} \). The main result of the paper is that the obtained system is integrable, in the sense that there is a sufficient number of invariant quantities which allow a separation of the variables and therefore integration in quadratures. The separating variables and first integrals are explicitly given.

*Lubomir Gavrilov*

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Shamolin, Maxim V. (RS-MOSC-MC)

★Dynamical pendulum-like nonconservative systems. (English summary)

The authors classify symmetric classes of rationalizable choice functions which possess the dictatorial property in terms of K. Arrow on the subsets of a finite set $A$ with cardinality $r$.

Fouad T. Aleskerov

This paper concerns a problem in nonlinear rigid body mechanics. Starting from the 3D generalization of an integrable planar problem, the author considers a rigid body interacting with the environment by means of a resisting (quasi-stationary) force. The body is axially symmetric with a planar frontal butt in the form of a two-dimensional disc and the interaction is concentrated in this part. The main result contained in this paper is the determination of the full sets of first integrals for the problem under investigation. The interesting feature is that some of the first integral contains transcendental functions of dependent variables expressed in terms of a finite combination of the elementary functions.

For further information pertaining to this item see [M. V. Shamolin, Autom. Remote Control 74 (2013), no. 10, 1771; MR3219867].

REVISED (August, 2016)
Shamolin, M. B. [Shamolin, Maxim V.] (RS-MOSC-IMC)

Erratum: “A new case of integrability in transcendental functions in the
dynamics of solid body interacting with the environment” [Autom. Remote
Control 8, 1378 (2013)] [MR3224103].


Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)

Some questions of qualitative theory in dynamics of systems with the variable
dissipation.


Summary: “In this work, we consider some problems of the qualitative theory of ordinary
differential equations; the study of dissipative systems, as well as variable dissipation
system considered below, which, in particular, arise in the dynamics of a rigid body in-
teracting with a medium and in the oscillation theory, depends on solutions of these
problems. We consider such problems as existence and uniqueness problems for traject-
cories having infinitely remote points as limit sets for systems on the plane, elements
of qualitative theory of monotone vector fields, and also existence problems for families
of long-period and Poisson stable trajectories. In conclusion, we study the possibility
of extending the Poincaré two-dimensional topographical system and the comparison
system to the many-dimensional case.”

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   fixed point in $\mathbb{R}^n$,” In: Abstracts of Sessions of the Workshop ”Topical Problems
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**American Mathematical Society**

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**Mathematical Reviews**

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MR3207076 | 35R35 35K25
Selivanova, N. Yu. (RS-AOS-TI);
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)
Quasi-stationary Stefan problem with values on the front depending on its geometry. (English summary)

Summary: "The problem presented below is a singular-limit problem of the extension of the Cahn-Hilliard model obtained via introducing the asymmetry of the surface tension tensor under one of the truncations (approximations) of the inner energy."

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10. O. Penrose and P. Fife, "On the relation between the standard phase-field model


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Selivanova, N. Yu. (RS-AOS-TI); Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)

Studying the interphase zone in a certain singular-limit problem. (English summary)


**References**

Selivanova, N. Yu. (RS-AOS-TI); Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)

Local solvability of a one-phase problem with free boundary. (English summary)

Summary: “A certain one-phase problem with free boundary is studied. The local (in time) solvability of this problem is proved; moreover, the general method elaborated is applied in a more concrete case. For this purpose, a new change of variables and the parametrization of the boundary are introduced, and the problem studied is reduced to a problem in a constant domain.”

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

Shamolin, M. V. [Shamolin, Maxim V.] (RS-AOS-MC)
A new case of integrability in the dynamics of a multidimensional rigid body in a nonconservative field. (Russian)

The dynamics of a multidimensional rigid body described by an analogue of Euler and Newton equations on $\text{so}(n) \times \mathbb{R}^n$ is considered. A new integrable case in this dynamics is described.

Igor N. Nikitin

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)
On integrability in dynamic problems for a rigid body interacting with a medium. (English summary)

This paper addresses the computation of first integrals for the differential equations describing the dynamics of a homogeneous axisymmetric rigid body with a disk-shaped frontal area moving in a resisting medium. In particular, the author considers the motion of a body acted upon by a follower force which is directed along the geometrical symmetry axis of the body. Two cases of a constant follower force are treated in detail; in particular, first integrals, in terms of either elementary transcendental functions or analytic functions, are found.

M. E. Sousa-Dias
A complete list of first integrals of the dynamic equations of motion of a four-dimensional rigid body in a nonconservative field in the presence of linear damping. (Russian)


A 4-dimensional homogeneous rigid body having a 2-dimensional disk as a flat front end face is considered. For the special case when the resistant force acting on the disk is concentrated on that part of the body surface that is shaped as a three-dimensional ball, the part of the equations of motion corresponding to the algebra so(4) is derived. In the case when the body has the dynamical symmetry property and an additional tracking force is applied, the complete list of nine invariant relations (first integrals) is given. Six of them are trivial and three others are transcendental functions in \( \mathbb{C} \) which can be represented in the form of finite combinations of the elementary functions.

*A. S. Sumbatov*

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Variety of the cases of integrability in dynamics of a symmetric 2D-, 3D- and 4D-rigid body in a nonconservative field. (English summary)


The author considers the cases of integrability in dynamics of two- and three-dimensional rigid bodies in a nonconservative force field and develops the idea of generalisation of the equations to the case of a four-dimensional rigid body in an analogous nonconservative force field. As a result of this generalization, he obtains a variety of cases of integrability for the problem of body motion in a resisting medium that fills the four-dimensional space in the presence of a certain tracing force that allows one to reduce the order of the general system of ODE in a methodical way.

*A. E. Zakrzhevskii*
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-NDM)
A new integrability case of equations of dynamics on the tangent bundle of a 3-sphere. (Russian)

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)
Comparison of complete integrability cases in dynamics of a two-, three-, and four-dimensional rigid body in a nonconservative field. (English summary)

In this paper the author investigates the Arnold-Liouville integrability for a dynamically symmetric four-dimensional rigid body having forward plane endwall which moves in a certain resistant media under action of non-conservative and tracing forces. Provided that the center mass of the body has always uniform rectilinear motion and that some quasi-stationary conditions take place, the complete set of first integrals is given for a plane-parallel motion of the body. The analogous results are obtained in the cases of three-dimensional space and, in the most complicated case concerned, the four-dimensional space of motion. It is proved that, in each case, one first integral is an analytic function of the phase variables and the other first integrals are transcendental (after the formal continuation to the complex domain).

A. S. Sumbatov

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.
Levi-Civita symbols, generalized vector products, and new integrable cases in mechanics of multidimensional bodies.


Two new cases of completely integrable dynamics of a 4-dimensional rigid body in a non-conservative force field of a special form are presented.

**Igor N. Nikitin**

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**References**


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-NDM)
The problem of the motion of a body in a resisting medium taking into account
the dependence of the force moment of resistance on the angular momentum.
(Russian. English, Russian summaries)
A new integrability case in the dynamics of a four-dimensional rigid body in a nonconservative field under linear damping. (Russian)


The paper deals with the dynamics of a four-dimensional rigid body described by the system of differential equations

$$\dot{\Omega} \Lambda + \Lambda \dot{\Omega} + [\Omega, \Omega \Lambda + \Lambda \Omega] = M,$$

where $\Omega \in \text{so}(4)$ is the matrix of the angular velocity, while $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_4)$ is the inertia tensor, $\lambda_i = \frac{1}{2}(I_1 + I_2 + I_3 + I_4) - I_i$. The main difference from the previously studied integrable cases is the presence of non-conservative external forces entering into the equations of motion through the matrix of the external momentum $M$. In the highly symmetric case $I_2 = I_3 = I_4$, the author gives a (rather complicated and artificial) choice of the momentum $M$ depending linearly on $\Omega$, which ensures the existence of a full set of integrals of motion.

Yuri B. Suris

A complete list of first integrals for the dynamic equations of motion of a rigid body in a resisting medium taking into account linear damping. (Russian. English, Russian summaries)


The present paper considers a new case of integrability in the problem of spatial rigid body motion in the presence of a nonconservative moment of forces. A nonconservative force field of action of the medium on the body is constructed taking into account the linear dependence of the field on the angular velocity.

Clementina D. Mladenova
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)

A new integrability case in the spatial dynamics of a rigid body interacting with a medium taking linear damping into account. (Russian)


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Trofimov, V. V. [Trofimov, Valerii Vladimirovich] (RS-MOSC); Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC)

Geometric and dynamical invariants of integrable Hamiltonian and dissipative systems. (Russian. English, Russian summaries)


This paper is related to the two D.Sc. theses of the authors on various aspects of the dynamics of integrable systems.

The first part of the paper is based on research carried out by Trofimov. In the first chapter a method for constructing completely integrable Hamiltonian systems on the coadjoint representation of Lie groups is proposed. Within this method new examples of completely integrable systems are constructed. This method makes it possible to prove the complete integrability of the equations, previously known as a multidimensional extension of the equations of magnetohydrodynamics. A theorem on the complete integrability of the Euler equations on tensor extensions of semisimple Lie algebras is proved. The second chapter is devoted to a geometric construction allowing one to classify Hamiltonian systems with first integrals. The construction mentioned is based on the extension of the Maslov class concept. Completely integrable systems with nontrivial generalized Maslov classes on the coadjoint orbits of Lie groups of small dimension are explored in Chapter 3.

The second part of the book is based on research carried out by Shamolin. Some classes of completely integrable non-conservative systems are investigated in Chapter 4. Systems under the action of non-conservative forces and variable dissipation are considered in Chapter 5. A system possessing a first integral which is a transcendental function of phase variables is pointed out. Some examples related to rigid body dynamics under the action of non-conservative forces are studied. Invariant indices characterizing countable sets of phase portraits are discussed. In Chapter 6, cases of the complete integrability of a four-dimensional dynamically symmetric top moving under the action of non-conservative forces are indicated.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

This paper deals with the plane-parallel motion of a rigid body interacting with a homogeneous stream of a resisting medium only through a forward flat site of its surface. The motion of the resisting medium is not studied. Gravity is considered to be small in comparison with the force of action of the medium. The considered model corresponds to a problem about putting homogeneous circular cylinders in water. The force action of the medium is defined on the basis of experimental data. The mathematical model for the problem is reduced to the equations of plane-parallel motion of a rigid body. The problem’s generalized coordinates are defined by the velocity vector and the magnitude of the angular velocity of the body. The force action of the medium is considered in the framework of a linearized model. The main objective of the research consists in studying the effect of the torque of the force of action of a medium. Further, conditions of the asymptotic stability of rectilinear translational braking are investigated. A multiparameter set of phase portraits in the space of quasi-velocities is obtained. Separately the problem about putting a hollow cylinder in water is considered with the objective of defining relations of its mass-geometrical parameters, which would ensure the stability of the braking of a translational motion of such a
cylinder in water. At the end of the paper, the author adduces reasons concerning features of carrying out natural experiments.

O. Christov

The motion of a four-dimensional rigid body in a nonconservative field is considered. This study is a continuation of previous investigations of the author in lower dimensions. Under certain assumptions on the nature of the nonconservative field, the equations of motion of the rigid body are derived and then analyzed.

For the dynamically symmetric rigid body a complete list of first integrals is given. Thereby, a new integrable case is found.

O. Christov
the global type of the unperturbed phase portrait (these changes occur infinitely many
times).
Classification of complete integrability cases in the dynamics of a symmetric four-dimensional rigid body in a nonconservative field. (Russian. Russian summary)


Summary (translated from the Russian): “This paper is a relatively final result in the investigation of the equations of motion of a dynamically symmetric four-dimensional rigid body in two logically possible cases of its tensor of inertia in a nonconservative force field. The form of the force field considered is taken from the dynamics of real three-dimensional rigid bodies interacting with a medium.”

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

Averaging operators and real equations of fluid mechanics. (Russian. Russian summary)


Summary (translated from the Russian): “We discuss pseudodifferential operators that appear in real equations of continuum mechanics.”

References

The article concerns methodological principles of the theory of mechanical systems. The authors show that the adequate description of multiphase multivelocity flows must use not differential but pseudodifferential equations and these equations must be hyperbolic.

Yu. V. Egorov

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.
New integrability cases in the three-dimensional dynamics of a rigid body. (Russian)


From the text (translated from the Russian): “The results of this paper are based on an investigation of ours of a problem of the motion of a rigid body in a resisting medium [Methods for the analysis of dynamical systems with variable dissipation in the dynamics of a rigid body (Russian), Ékzamen, Moscow, 2007; Fundam. Prikl. Mat. 14 (2008), no. 3, 3–237; MR2482029], where we dealt with first integrals of dynamical systems with nonstandard properties. Specifically, the integrals were neither analytical nor smooth, and for certain sets, they were even discontinuous. These properties allowed us to thoroughly analyze all phase trajectories and to indicate the properties that possessed 'structural stability' and were preserved for systems of more general form with certain nontrivial symmetries of hidden type. Therefore, it is of interest to investigate a sufficiently large class of systems with similar properties, in particular, those involving the dynamics of a rigid body interacting with a medium. In this paper, we present new integrability cases in the problem of the three-dimensional dynamics of a rigid body in a resisting medium.”

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*

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Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)


Summary: “The paper discusses a nonlinear model that describes the interaction of a rigid body with a medium and takes into account (based on experimental data on the motion of circular cylinders in water) the dependence of the arm of the force on the normalized angular velocity of the body and the dependence of the moment of the force on the angle of attack. An analysis of plane and spatial models (in the presence or absence of an additional follower force) leads to sufficient stability conditions for translational motion, as one of the key types of motions. Either stable or unstable self-oscillation can be observed under certain conditions.”
On the integrability in elementary functions of some classes of nonconservative dynamical systems. (Russian. Russian summary)

Summary (translated from the Russian): “The results in this paper are based on the investigation of the applied problem of the motion of a rigid body in a resisting medium [V. A. Samsonov, B. Ya. Lokshin and V. A. Privalov, “Qualitative analysis” (Russian), Sci. Rep. Inst. Mech. Moscow State Univ. No. 3425, Moskov. Gos. Univ., Moscow, 1985; per bibl.; V. A. Samsonov et al., “Mathematical modeling in the problem of the deceleration of a body in a resisting medium in the case of a jet flow around the body” (Russian), Sci. Rep. Inst. Mech. Moscow State Univ. No. 4396, Moskov. Gos. Univ., Moscow, 1995; per bibl.], in which complete lists of transcendental first integrals expressed in terms of a finite combination of elementary functions were obtained. This made it possible to thoroughly analyze all the phase trajectories and to determine which of their properties possess structural stability and which are preserved in systems of more general form. The complete integrability of such systems is related to hidden symmetries. Therefore, it is of interest to study sufficiently wide classes of dynamical systems that have similar hidden symmetries.

“As is known, the concept of integrability is, in general, fairly broad. Thus, it is necessary to take into account in what sense it is understood (a criterion according to which one can conclude that the structure of the trajectories of the dynamical system considered is especially ‘attractive and simple’) in the function classes in which the first integrals are sought, etc.

“In this paper, we use an approach in which the first integrals are transcendental functions, and in fact elementary. Here transcendence is understood not in the sense of elementary functions (for example, trigonometric) but in the sense that they have essentially singular points (according to the classification used in the theory of functions of one complex variable in the case when the function has essentially singular points). In this connection, it is necessary to continue them formally to the complex plane. As a rule, such systems are strongly nonconservative.”

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In this investigative review the authors aim to define groups of colors, elaborating on what kind of groups can belong to such color groups and how they should differ from the graded subgroups. Much emphasis is placed on the Yang-Baxter symmetry, which has been shown to play a crucial role in describing the notion of a true color group. The central concept is explained in a systematic way through several definitions, statements and their proofs. The notion of the color group is shown to be related to the grading over the algebra, which in turn is linked also to the symmetry and the solution of the Yang-Baxter relation. The subtle difference between the grading of a group and a colored group is explained by introducing the notion of bicharacter. It is emphasized through several steps that, to every grading element $g$, a color can be assigned constituting a set of equivalent $g$-grading with the bicharacter depending only on the color group and not on the empty part of the grading. As an illuminating example, the well-known Clifford algebra is shown to be a color algebra of a color group.

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New cases of complete integrability in the dynamics of a dynamically symmetric four-dimensional rigid body in a nonconservative field. (Russian)


Two conditional integrable cases are constructed in the dynamics of a 4-dimensional axisymmetric rigid body moving under the action of a resistance-like follower-force applied to a certain specially chosen point on the body. Two types of axial symmetry are considered, in which the inertia matrix has three (or two pairs of) equal eigenvalues. The dynamics is shown to be integrable on the intersection of three (or two) invariant hyperplanes of the space of angular velocities.

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Summary: “This work is devoted to the development of qualitative methods in the theory of nonconservative systems that arise, e.g., in such fields of science as the dynamics of a rigid body interacting with a resisting medium, oscillation theory, etc. This material can attract the interest of specialists in the qualitative theory of ordinary differential equations, in rigid body dynamics, as well as in fluid and gas dynamics since the work uses the properties of motion of a rigid body in a medium under the streamline flow around conditions.

“The author obtains a full spectrum of complete integrability cases for nonconservative dynamical systems having nontrivial symmetries. Moreover, in almost all cases of integrability, each of the first integrals is expressed through a finite combination of elementary functions and is a transcendental function of its variables, simultaneously. In this case, the transcendence is meant in the complex analytic sense, i.e., after the continuation of the functions considered to the complex domain, they have essentially singular points. The latter fact is stipulated by the existence of attracting and repelling limit sets in the system considered (for example, attracting and repelling foci).

“The author obtains new families of phase portraits of systems with variable dissipation on lower- and higher-dimensional manifolds. He discusses the problems of their absolute or relative roughness. He discovers new integrable cases of the rigid body motion, including those in the classical problem of motion of a spherical pendulum placed in the over-running medium flow.”

A. P. Sadovskiǐ

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**From the text (translated from the Russian):**

“The results of this paper are due to a previous investigation of the applied problem of the motion of a rigid body in a resisting medium [V. A. Samsonov and M. V. Shamolin, *Vestnik Moskov. Univ. Ser. I Mat. Mekh.* 1989, no. 3, 51–54, 105; MR1029730] in which a transcendental integral expressed in terms of elementary functions was obtained for a particular case. This made it possible to carry out a complete analysis of phase trajectories and to indicate those properties that were ‘robust’ and preserved for some more general systems. The integrability of the system in [op. cit.] is related to latent symmetries. Therefore, it is of interest to study sufficiently large classes of dynamical systems with such latent symmetries.”
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-IMC)

A three-parameter family of phase portraits in the dynamics of a rigid body interacting with the medium. (Russian)


Summary (translated from the Russian): “We construct two- and three-dimensional nonlinear models of the action of a medium on a rigid body, which take into account the dependence of the arm of the force on the reduced angular velocity of the body when the moment of force is also a function of the angle of attack. We find new cases of complete integrability in elementary functions, which makes it possible to discover qualitative analogies between the motions of free bodies in a resisting medium and the oscillations of bodies that are partially fixed and immersed in a flow of the medium. We show that if the additional damping action of the medium on the body that occurs in the system is significant, then it is possible to stabilize the rectilinear translational deceleration of the body when it moves with finite angles of attack. In this connection, the question of the roughness of the description of this phenomenon is of current interest: a finer property of relative roughness is discovered in the investigation of reduced dynamical systems.”

Shamolin, M. V. [Shamolin, Maxim V.]

New integrable cases in the dynamics of a body interacting with a medium taking into account the dependence of the resistance force moment on the angular velocity. (Russian. Russian summary)


Summary (translated from the Russian): “We construct two- and three-dimensional nonlinear models of the action of a medium on a rigid body, which take into account the dependence of the arm of the force on the reduced angular velocity of the body when the moment of force is also a function of the angle of attack. We find new cases of complete integrability in elementary functions, which makes it possible to discover qualitative analogies between the motions of free bodies in a resisting medium and the oscillations of bodies that are partially fixed and immersed in a flow of the medium. We show that if the additional damping action of the medium on the body that occurs in the system is significant, then it is possible to stabilize the rectilinear translational deceleration of the body when it moves with finite angles of attack. In this connection, the question of the roughness of the description of this phenomenon is of current interest: a finer property of relative roughness is discovered in the investigation of reduced dynamical systems.”

A˘ıdagulov, R. R. (RS-MOSCM); Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSCM)

Manifolds of continuous structures. (Russian. Russian summary)


Summary (translated from the Russian): “In modern mathematics, the theory of categories and functors provides a language for describing arbitrary sets. Its main distinguishing feature is that instead of considering an individual set with some given structure, one considers all identically structured sets simultaneously. In this paper, we
describe systems by categories. The system itself is a category that consists of a class of objects and a class of morphisms. The axiomatics of the mathematical structure defining the category distinguishes the given system from among other systems, and the objects of the category model the state of systems.”

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MR2342524 (2009g:74015) 74B05 35Q72

Aidagulov, R. R. (RS-MOSCM); Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSCM)

A general spectral approach to the dynamics of a continuous medium.

(Russian. Russian summary)


In this paper the authors come to the conclusion that the system of conservation mass and moment is not suitable for the calculation of resonances and instabilities in a medium. In the opinion of the authors, the homogenization method is also not useful in such problems. A system of pseudo-differential equations with more precise dispersion relations in a medium is obtained by varying the waves and amplitudes description of the medium. In accord with this method the authors give a pseudo-differential system
for rarefied gas dynamics, for a pseudo-differential version of Hooke’s law, and discuss the
possibility of definition of the medium by means of spectral properties of the medium.

Mikhail P. Vishnevskii

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.
Archimedean uniform structures. (Russian. Russian summary)


Summary (translated from the Russian): “In mathematics, the concept of the Archimedean property is used in connection with two different objects: orderings of groups and valuations of rings. In both cases, one can define a topology on these objects and even a uniform structure; in the first case, an interval topology, and in the second, a certain valuation. It turns out that these two uses of the term Archimedean property and the somewhat regrettable term ‘topological group without small subgroups’ are special cases of the concept of the Archimedean property of a topological group.”

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17. L. S. Pontryagin, Continuous Groups [in Russian], Moscow (1973). MR0357673
20. A. Frölicher and V. Bucher, Differential Calculus in Vector Spaces without Norm
References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.
A case of complete integrability in the dynamics on the tangent bundle of a two-dimensional sphere. (Russian)

From the text (translated from the Russian): “We propose a new approach that enables us to obtain integrable cases in the dynamics of a free rigid body, namely, to integrate dynamic equations on the space $\mathbb{R}^3 \times \mathfrak{so}(3)$, in a nonconservative (dissipative) force field.”

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

Comparison of Jacobi-integrable cases of two- and three-dimensional motions of a body in a medium in the case of a jet flow. (Russian. Russian summary)

Summary (translated from the Russian): “We show the complete integrability of the plane problem of the motion of a rigid body in a resisting medium under jet flow conditions, when one first integral, which is a transcendental function of quasi-velocities (in the sense of the theory of functions of a complex variable with essentially singular points), exists in the system of equations of motion. It is assumed that the entire interaction of the medium with the body is concentrated on a part of the surface of the body that has the shape of a (one-dimensional) plate. We generalize this plane problem to the three-dimensional case, where a complete set of first integrals exists for the equations of motion: one analytic, one meromorphic, and one transcendental. Here we assume that the entire interaction of the medium with the body is concentrated on part
of the surface of the body that has the shape of a flat (two-dimensional) disk. We also attempt to construct a generalization of the ‘low-dimensional’ cases to the dynamics of a so-called four-dimensional rigid body whose interaction with a medium is concentrated on a part of the (three-dimensional) surface of the body that has the shape of a (three-dimensional) sphere. In this case, the angular velocity vector is six-dimensional, while the velocity of the center of mass is four-dimensional.”

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC)
On an integrable case of equations of dynamics on \( \text{so}(4) \times \mathbb{R}^4 \). (Russian)

The four-dimensional analog of the problem on the motion of a rigid body under the action of resistance forces with variable dissipation and of a servo-constraint is studied. The rotational part of the equations of motion is considered under the assumption that the body is dynamically symmetric. It is shown that under appropriate conditions the equations of motion possess an invariant surface. For the motions restricted to this surface transcendental first integrals are indicated.  

References

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

**Citations**

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From the text (translated from the Russian): “Because of its complexity, the problem of the motion of a rigid body in an unbounded medium requires the introduction of simplifying restrictions. The main goal is to introduce hypotheses that allow one to study the motion of a rigid body separately from the motion of the medium in which the body is located. On the one hand, a similar approach was taken in the classical Kirchhoff problem of the motion of a body in an unbounded ideal incompressible fluid which is at rest at infinity and which undergoes irrotational motion. On the other hand, it is clear that the aforementioned Kirchhoff problem does not exhaust the possibilities of this type of modeling.

“In this paper, we consider the possibility of transferring the results of the dynamics of the plane-parallel motion of a homogeneous axisymmetric rigid body interacting at its front circular face with a uniform flow of a resisting medium to the case of three-dimensional motion. Here, unlike in previous papers on the modeling of the interaction between a medium and a rigid body, we take into account the effects of the so-called rotational derivatives of the moment of hydroaerodynamic forces with respect to the components of the angular velocity of the rigid body itself.”

**Citations**

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Summary (translated from the Russian): “We refine Conway’s algorithm for computing prime numbers. In the course of analyzing it, we establish that some numbers obtained using it are erroneous. Further investigation leads to the determination of tabularly computable functions and the establishment of the equivalence of this class of functions
and the class of recursive functions.”

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From the text (translated from the Russian): “We carry out a complete qualitative analysis of a model version of the plane-parallel motion of a body in a resisting medium under a jet-flow condition for the oscillatory domain of the phase space of the dynamic equations. We assume that the velocity of the center of the plate through which the body interacts with the medium remains constant throughout the motion. As was shown earlier, the dynamical system in the space of quasivelocities is relatively structurally stable (robust with respect to physically admissible classes of dynamical systems). We consider an additional qualitative integration of the kinematic relations. We study the properties of the solutions corresponding to the oscillatory domain: the properties of the asymptotes associated with the motion of the rigid body, various equivalence relations in the trajectory space, topological analogies, and mechanical interpretations of asymptotic motions. We study the local property of the asymptote.”

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<td>MR2082898 (2005j:70014) 70E99 34C40 37N05 70K05</td>
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In this paper the author applies a rather simplified mechanical model for the study and explanation of nontrivial effects arising in the plane-parallel and spatial motions of a rigid body in a resisting medium. He assumes that the interaction of the medium with the body is concentrated at the front part of the body’s surface, which has the form of a flat plate.

The paper consists of six chapters. In the first chapter the author considers several forms of nonlinear dynamical systems describing the motion of a body in a medium. In the next four chapters the author develops the standard methods of the qualitative theory of ordinary differential equations and successfully uses them for investigation and classification of phase trajectories of dissipative systems of special types. In the final
chapter he studies the stability of the translational deceleration and self-oscillations of the moving body in the presence of a linear damping momentum.  

A. Yu. Savchenko

References

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301. V. V. Stepanov, *A Course in Differential Equations* [in Russian], Fizmatgiz, Moscow (1959).


316. I. N. Vrublevskaya, "On geometrical equivalence of trajectories and half-trajectories
of dynamical systems,” *Mat. Sb.*, **42** (1947).


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.
linear nonhomogeneous nonautonomous system. We study in detail the case $n = 4$.

The paper presents the results of an analysis of the aircraft control system diagnostics problem. The motion of the control system is described by nonlinear ordinary differential equations. First, the diagnostics problem is considered in the case of exact trajectorial measurements. Second, trajectorial measurements corrupted by normal white noise with zero mean value and bounded spectrum are analyzed. Third, trajectorial measurements are investigated in the case when their errors are normal random variables whose absolute values are bounded by a certain function of time. Solving the diagnostics problem allows one to repair the control system and isolate the trouble. To solve the diagnostics problem, the following data are used: the mathematical model of the motion of the object, the bounded domain of its initial conditions and the list of models describing the motion of this object with a fault. A statistical-modelling method for selection of the checking surface is proposed and checking surface accessibility conditions are investigated. A description of diagnostic techniques involving selection of the checking surface is proposed. Particular attention is given to the case of linear systems. The author emphasizes the deep differences of his approach to the diagnostics problem in comparison with other works. Namely, the diagnostics problem is considered in terms of a classification of malfunctions. The mathematical modelling of malfunctions is presented in this context. General statements of the differential diagnostics problem are discussed in detail. Various extensions of the diagnostics theorem are described.

Valery I. Korobov

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.
New integrable cases and families of portraits in the plane and spatial dynamics of a rigid body interacting with a medium.

Dynamical systems, 12.


Several models are considered for the motion of a rigid body in a resisting medium (like a gas or a fluid). Probably the best known model in this area is the Kirchhoff system for the motion of a rigid body in an ideal fluid. In the present paper somewhat different physical assumptions are used. The resulting differential equations are nonlinear systems of dimension 2, 3, or 4. Their phase portraits are analysed, and several integrable cases are pointed out.

**Yuri B. Suris**

References

4. A. A. Andronov and E. A. Leontovich, "Bifurcation of limit cycles from a structurally unstable focus or center and from a structurally unstable limit cycle," *Mat. Sb.*, 40, No. 2 (1956). MR0085413
5. A. A. Andronov and E. A. Leontovich, "On the bifurcation of limit cycles from a separatrix loop and from the separatrix of the equilibrium state of the saddle-node type," *Mat. Sb.*, 48, No. 3 (1959). MR0131612
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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.
MR1930111 (2003g:70008) 70E17
Georgievskii, D. V. [Georgievskii, Dmitri V.] (RS-MOSC);
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC)
Generalized dynamic Euler equations for a rigid body with a fixed point in $\mathbb{R}^n$. (Russian)

The authors continue the investigations they began in an earlier paper [Dokl. Akad. Nauk 380 (2001), no. 1, 47–50; MR1867984] in which they studied the kinematics and mass geometry of an $n$-dimensional rigid body with a fixed point in $\mathbb{R}^n$. The present paper contains a derivation of the generalized dynamic Euler equations for this problem. Using the representation of a differential equation that generalizes the classical law of the change in angular momentum of a body in terms of dual tensors, the authors obtain generalized dynamic Euler equations. They consider in detail the case when there are no external forces. For this case they show that the number of independent first integrals is less than the number of components of angular velocity by the value $\frac{1}{2}(n - 2)(n - 1)$.

**Gennady Victorovich Gorr**

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MR1919087 (2004j:37161) 37N05 34C05 37G15 70E15 70K50
Shamolin, M. V. [Shamolin, Maxim V.]
Some questions of the qualitative theory of ordinary differential equations and dynamics of a rigid body interacting with a medium.
Dynamical systems, 10.

This paper is a translated version of the Russian original; it contains several mistakes and typos. The paper treats the problem of appearance (or disappearance) of limit cycles for a vector field on $\mathbb{R}^2$. A survey is offered and old results such as the Hopf bifurcation scenario are recalled. The author shows how such bifurcations occur in a system describing the motion of a rigid body interacting with a medium. Stokes’ theorem (referred to as the Gauss-Ostrogradskiǐ formula or the Green formula in the present paper), as well as the Poincaré-Bendixson theorem, are used repeatedly.

**Vincent Naudot**

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**References**

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14. V. V. Golubev, Lectures on the Integration of the Equations of Motion of a Heavy Rigid Body Around a Fixed Point [in Russian], Gostekhizdat, Moscow (1953). MR0061942


On the integration of some classes of nonconservative systems. (Russian)


Some systems of a nonconservative type encountered in the dynamics of rigid bodies interacting with a medium are considered.

Proposition. The \((2n - 1)\)-parametric set of systems of equations on the plane \(\mathbb{R}^2(x, y)\),

\[
\dot{x} = ax + by + \sum_{i=1}^{2n-1} \delta_i x^{2n-i} y^{i-1},
\]

\[
\dot{y} = cx + dy + \sum_{i=1}^{2n-1} \delta_i x^{2n-(i+1)} y^i,
\]

has a (generally speaking, transcendental) first integral, expressed via elementary functions.

L. M. Berkovich

References


Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)
Integrability cases for equations of the three-dimensional dynamics of a rigid body. (Russian. English, Ukrainian summaries)

Summary: “A dynamic model of the interaction of a rigid body with a resisting medium under conditions of a jet flow is considered. This model allows one to extend the results for the corresponding problems from plane dynamics of a rigid body interacting with the medium and to obtain their three-dimensional analogues, as well as to establish the integrability in the sense of Jacobi of the new cases. Thus, the integrals in some cases can be expressed in terms of elementary functions. The classical problem of a spherical pendulum in a jet flow and that of the motion of a three-dimensional body with a servoconstraint are proved to be integrable. Mechanical and topological analogues of these problems are presented.”

Shamolin, M. V. [Shamolin, Maxim V.]
Complete integrability of equations of motion of a spatial pendulum in an incident medium flow. (Russian. Russian summary)

Summary (translated from the Russian): “Previously, we considered the problem of a plane pendulum in an incident medium flow. In the present paper we construct a generalization of this problem to the spatial case. We establish the complete integrability in the sense of Jacobi of this problem. In the plane case there sometimes exists a single transcendental first integral expressed in terms of elementary functions, but in the
spatial case there can be several such integrals (under certain conditions)."

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**MR1867984 (2003a:70002) 70B10 70E17**

Georgievskii, D. V. [Georgievskii, Dimitri V.] (RS-MOSC);
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC)

Kinematics and mass geometry of a rigid body with a fixed point in \( \mathbb{R}^n \).

(Russian)


The authors consider the kinematics and mass geometry of a rigid body with a fixed point in an \( n \)-dimensional space. Using the generalized Euler formula and the angular velocity tensor they determine the velocities of the points of the body. The angular velocity tensor of \((n - 2)\)nd rank is associated with the dual angular velocity tensor of the second rank. The authors use the generalized Rival formula to determine the accelerations of the points of the body. In the case of hyperplane motion, they determine the components of the angular velocity tensor and the components of the dual angular velocity. They find relations for the angular momentum of the body for kinetic energy. They show that the mass geometry of an \( n \)-dimensional rigid body is determined by the second-order symmetric inertia tensor. For \( n = 3 \), the tensor \( I^{(2)} \), introduced to define the angular momentum, and the tensor \( J^{(2)} \), characterizing the kinetic energy, coincide and are the conventional inertia tensor in \( \mathbb{R}^3 \). The results obtained are only of theoretical interest.

*Gennady Victorovich Gorr*

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**MR1844242 (2002d:90024) 90B25**

Borisenok, I. T.; Shamolin, M. V. [Shamolin, Maxim V.]

Solution of the differential diagnostic problem by the statistical testing method.

(Russian. Russian summary)


Summary (translated from the Russian): “The differential diagnostic problem for the functional state of control plants having a modular structure and a finite set of possible failures can be reduced to two independent sequentially solvable problems: the control problem, i.e., the establishment of a criterion for the presence of failure in the system, and the diagnostic problem, i.e., the identification of the failure. The criterion for failure in the system can be the plant trajectory leaving some prespecified surface. The failure can occur at any previously unknown instant during the motion of the plant and at any point within the specified surface. The diagnostic problem can be solved by tracking the trajectory of the plant after its departure from the control surface.
We give a solution to the differential diagnostic problem for dynamical control systems in the case of trajectory measurements with noise, starting from general probabilistic considerations.

Citations

From References: 16 From Reviews: 1

MR1833828 (2002c:70005) 70E15
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-IMC)
Integrability in the sense of Jacobi in the problem of the motion of a four-dimensional rigid body in a resisting medium. (Russian)

From the text (reviewer’s translation): “This paper is devoted to studying the motion of a so-called four-dimensional rigid body that interacts with a resisting medium according to ‘streamline flows’. It is assumed that all the interactions of the rigid body with the medium are concentrated on the part of the surface of the body (three-dimensional) that has the shape of a ball (three-dimensional).”

The results are as in the title.

J. S. Joel

Citations

From References: 9 From Reviews: 0

MR1777365 (2002d:34049) 34C05
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC)
On limit sets of differential equations near singular equilibrium points. (Russian)

For the third-order system
\[
\alpha' = -z_2 + \sigma(z_1^2 + z_2^2) \sin \alpha + \sigma n_0^2 \sin \alpha \cos^2 \alpha + (B \sin \alpha \cos \alpha) / m,
\]
\[
z_2' = n_0 \sin \alpha \cos \alpha - z_2 \psi(\alpha, z_1, z_2) - z_1^2 \cos \alpha / \sin \alpha,
\]
\[
z_1' = -z_1 \psi(\alpha, z_1, z_2) + z_1 z_2 \cos \alpha / \sin \alpha,
\]
where
\[
\psi(\alpha, z_1, z_2) = -\sigma(z_1^2 + z_2^2) \cos \alpha + \sigma n_0^2 \sin^2 \alpha \cos \alpha - (B \cos^2 \alpha) / m,
\]
\[
\sigma, n_0, B, m > 0,
\]
the author proves the existence of an attracting limit cycle in the spherical layer \(\Pi(0, \pi) = \{(\alpha, z_1, z_2) \in \mathbb{R}^3 : z_1 > 0, 0 < \alpha < \pi\}\).

A. P. Sadovskii

References

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1947 (Russian); French original in Œuvres de Henri Poincaré, vol. 1, Guathier-Villars, Paris 1928.


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)
A new family of phase portraits in the three-dimensional dynamics of a rigid body interacting with a medium. (Russian)


The three-dimensional dynamics of a rigid body interacting with a viscous medium has been given an acceptable qualitative description in only the very simplest, most simplified situations. The dynamical model used in the present paper is rather simple and enables the author to form the equations of motion of the body without preliminary computations of detailed characteristics of the motion of the medium (for more details see the book by B. Ya. Lokshin, V. A. Privalov, and V. A. Samsonov [Vvedenie v zadachu o dvienii tela v soprotivlwe$ is srede (Introduction to the problem of the motion of a body in a resisting medium), Izdat. Mosk. Gos. Univ., Moscow, 1986; per bibl.]). In this paper the author attempts to generalize to the three-dimensional case a number of his results obtained for the plane-parallel dynamics of a body in a resisting medium. The correctness of such a generalization was discussed previously by the author [Izv. Ross. Akad. Nauk Mekh. Tverd. Tela 1997, no. 2, 65–68; RZhMat 1997:11 B291].

Thus, the author studies the fast motion of a dynamically symmetric body undergoing conditions of stream flow. The interaction of the medium with the body is concentrated at the bow (front part) of the surface of the body, which has the form of a flat disk. The author writes down the equations of motion of the body, and by excluding the cyclic variables he distinguishes an independent subsystem of three autonomous equations. Then he studies the singular points of the vector field on a three-dimensional noncompact (open) manifold. The analysis of the limit sets and the separatrices allows him to describe qualitatively the phase topology of the reduced system. Then the author introduces an
index that characterizes the behavior of the separatrices of the trajectories, and using this index he classifies a countable number of topologically distinct phase portraits that occur in the given problem.

Igor Gashenenko

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MR1806854  90B25
Borisenok, I. T.; Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC)
Solution of a problem of differential diagnostics. (Russian. English, Russian summaries)
New computer technologies in control systems (Russian) (Pereslavl'-Zalesskii, 1996).

{For the collection containing this paper see MR1806845}

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MR1741681  (2000j:37021)  37C20  34D30  70E99  70K99
Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC)
Robustness of dissipative systems and relative robustness and nonrobustness of systems with variable dissipation. (Russian)

From the text (translated from the Russian): “We present a brief survey of problems of relative structural stability (relative robustness) of dynamical systems considered not on the entire space of dynamical systems but only on some subspace of it [M. V. Shamolin, Uspekhi Mat. Nauk 51 (1996), no. 1(307), 175–176; MR1392692].”

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MR1806845  90B25
Borisenok, I. T.; Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC)
Solution of a problem of differential diagnostics. (Russian. English, Russian summaries)
New computer technologies in control systems (Russian) (Pereslavl'-Zalesskii, 1996).

Citations  From References: 1 From Reviews: 0

Citations  From References: 8 From Reviews: 0

Citations  From References: 20 From Reviews: 0
New integrable, in the sense of Jacobi, cases in the dynamics of a rigid body interacting with a medium. (Russian)


From the text (translated from the Russian): “The dynamic model of the interaction of a rigid body with a resisting medium under jet flow conditions that is considered not only allows us to successfully transfer the results of corresponding problems from the two-dimensional dynamics of a rigid body interacting with a medium and to obtain their three-dimensional analogues, it also reveals new Jacobi-integrable cases. Here the integrals can sometimes be expressed in terms of elementary functions. We demonstrate the integrability of the classical problem of a spherical pendulum submerged in an incident flow of a medium and the problem of the three-dimensional motion of a body in the presence of a servoconstraint. We also give mechanical and topological analogues of the latter two problems.”

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC-MC)

On integrability in transcendental functions. (Russian)

The problem of integrability of systems of ordinary differential equations in transcendental functions is discussed in this paper. Shamil Makhmutov

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC)

Spatial Poincaré topographical systems and comparison systems. (Russian)

The notions of the Poincaré topographical system, the characteristic function and the comparison system are generalised for the higher-dimensional case. Theorem. Assume that in the 1-connected domain $D \subset \mathbb{R}^n$ containing a unique singular point $x_0$ of the smooth vector field $v$, there exists the hypersurface $\Gamma \ni x_0$, $\Gamma \cap \partial D = \gamma$ such that there exists a Poincaré topographical system, having a center at $x_0$ and defined by a smooth function $V$, extended along $\Gamma$ up to $\gamma$, filling the domain $K \subseteq D$ and such that...
\((v, \text{grad } V)|_{\mathbb{R}^n} \geq 0\) in \(K\). Then in the domain \(D\) there is no closed curve consisting of the trajectories of the vector field \(v\) and intersecting \(\Gamma\). Applications to the center-focus problem are discussed.  

\emph{Natalia Borisovna Medvedeva}
For example, the author presents a two-parameter family of dynamical systems with a countable set of topologically different phase portraits.

Igor Gashenenko

Shamolin, M. V. [Shamolin, Maxim V.] (RS-MOSC)

Determination of relative robustness and a two-parameter family of phase portraits in the dynamics of a rigid body. (Russian)


The author gives a definition of the relative robustness of a system of differential equations that differs from previously used definitions. It contains two main points: sufficient smallness of the homeomorphism that produces the equivalence, and $C^1$-topology in the space of vector fields. As an example, the author considers a problem that describes the dynamics of a rigid body interacting with a medium. He proves a theorem on absolute robustness, from which it follows that there exists a two-parameter family of phase portraits in which a degenerate transition occurs in the passage from one topological portrait type to another. It should be noted that the space in which the system is absolutely robust has finite measure, while the space in which the system is a system of the first degree of robustness has measure zero in the original space.

Gennady Victorovich Gorr

Shamolin, M. V. [Shamolin, Maxim V.]

★Relative structural stability of dynamical systems in the problem of the motion of a body in a medium. (Russian)


{For the collection containing this paper see MR1809235}
A new two-parameter family of phase portraits in the problem of the motion of a body in a medium. (Russian)


The paper deals with the Kirchhoff problem on the motion of a rigid body in an infinite ideal incompressible fluid medium. The author considers a sixth order dynamic system from which a second order subsystem splits off. The complete topological classification of phase portraits is carried out and a two-parameter family of phase portraits consisting of an uncountable set of topologically distinct phase portraits is isolated. 

V. A. Sobolev
MR1293942 (95e:34036)  34C35  34C99
Shamolin, M. V. [Shamolin, Maxim V.]
Existence and uniqueness of trajectories that have points at infinity as limit sets
for dynamical systems on the plane. (Russian. Russian summary)

Summary (translated from the Russian): “We consider dynamical systems on the plane,
cylinder and sphere. For some classes of systems we prove the existence and uniqueness
of trajectories going out to infinity in the plane. For one-parameter systems of equations
having monotonicity properties on two-dimensional oriented surfaces, we examine the
problem of the existence and uniqueness of limit sets and their monotone dependence
on the parameters.”

MR1258007 (94i:70027)  70K99  34C99  70K20  76B05
Shamolin, M. V. [Shamolin, Maxim V.]
Phase portrait classification in a problem on the motion of a body
in a resisting medium in the presence of a linear damping moment.
(Russian. Russian summary)

Summary (translated from the Russian): “We present a qualitative analysis of a dynamical system that describes a model version of the problem of the plane-parallel motion of a body in a medium with jet or separated flow when the entire interaction of the medium with the body is concentrated on a part of the surface of the body having the form of a flat plate. The force of the interaction is directed along the normal to the plate, and the point of application of this force depends only on the angle of attack. A thrust force acts along the mean perpendicular to the plate, which ensures that the value of the velocity of the center of the plate remains constant throughout the motion. In addition, a damping moment, linear with respect to the angular velocity, is imposed on the body. We carry out the phase portrait classification of the system depending on the coefficient of the moment. We note the mechanical and topological analogies with a pendulum fixed in a flowing medium.”
MR1223987 (94b:34060) 34C99 34C05 34C25 76D99
Shamolin, M. V. [Shamolin, Maxim V.]
Application of the methods of Poincaré topographical systems and comparison systems in some concrete systems of differential equations. (Russian. Russian summary)

Summary (translated from the Russian): “We consider autonomous systems on the plane or a two-dimensional cylinder and study questions of the existence for various classes of systems of Poincaré topographical systems or more general comparative systems. As applications we consider dynamical systems that describe the plane-parallel motion of a body in a resisting medium as well as various model variants of it.”

Citations

From References: 8
From Reviews: 0

MR1293705 (95d:34060) 34C23 34C05 34C25 70E15
Shamolin, M. V. [Shamolin, Maxim V.]
Closed trajectories of various topological types in the problem of the motion of a body in a resisting medium. (Russian. Russian summary)

Summary (translated from the Russian): “We consider dynamical systems on a two-dimensional cylinder. We sharpen the theorems of Hopf, Bendixson and Dulac, after which it becomes possible to study closed trajectories of various topological types in connection with the problem of the motion of a body in a resisting medium. We give an example of a class of systems in the phase space of which there exists a continuum of closed trajectories of different types.”

Citations

From References: 9
From Reviews: 0

MR1214592 (93k:70028) 70H05 34C05 34C25 58F40 70E15
Shamolin, M. V. [Shamolin, Maxim V.]
On the problem of the motion of a body in a resistant medium. (Russian. Russian summary)

Summary (translated from the Russian): “We continue a qualitative analysis of a model variant of the interaction of a body with a resistant medium. Under the assumption that the motion is plane-parallel we completely analyze the case of constant velocity of the center of mass. We prove the presence of nonisolated periodic solutions, the absence of limit cycles and transcendental integrability, and present necessary and sufficient
conditions for expressing the integral in terms of elementary functions.”